Math 307 Homework
October 21, 2015

1. (a) Verify that

$$\mathcal{B} = \left(\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \right)$$

is an orthonormal basis of

$$U = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \,\middle|\, x + y + z = 0 \right\}.$$

(b) Find the matrix with respect to $\mathcal{B}$ of the linear map $\boldsymbol{T} : U \to U$ given by

$$\boldsymbol{T} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y \\ z \\ x \end{bmatrix}.$$

2. A function of the form

$$f(\theta) = \sum_{k=1}^{n} a_k \sin(k\theta) + \sum_{\ell=0}^{n} b_\ell \cos(\ell\theta)$$

for $a_1, \ldots, a_n, b_0, \ldots, b_m \in \mathbb{R}$ is sometimes called a **trigonometric polynomial**. If we put the inner product

$$\langle f, g \rangle = \int_0^{2\pi} f(\theta) g(\theta) \, d\theta$$

on the space of trigonometric polynomials, find the norm of f as written above.

Hint: You should be able to do this without calculating any integrals — most of the work has already been done in Section 3.2 of the book!