

Math 307 Homework
March 23, 2015

1. Apply the Gram–Schmidt process to $(1, x, x^2, x^3)$ to produce an orthonormal basis of $\mathcal{P}_3(\mathbb{R})$ with respect to the inner product

$$\langle p, q \rangle = \int_{-1}^1 p(x)q(x) dx.$$

Warning: This is a different inner product than in the example on p. 181.

Hint: It will simplify things to observe beforehand that

$$\int_{-1}^1 x^k dx = \begin{cases} 0 & \text{if } k \text{ is odd,} \\ \frac{2}{k+1} & \text{if } k \text{ is even.} \end{cases}$$

2. Compute the QR decomposition of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.