

Math 307 Homework  
November 2, 2015

1. Let  $V$  and  $W$  be finite-dimensional inner product spaces, and let  $T \in \mathcal{L}(V, W)$ . Show that

$$U := \{v \in V : \|Tv\| = \|T\| \|v\|\}$$

is a subspace of  $V$ .

*Hint:* Remember that if you have two non-zero vectors  $v_1, v_2$ , you can write  $v_2 = av_1 + bu$ , where  $\langle u, v_1 \rangle = 0$ .

2. Prove that if the subspace  $U$  of  $V$  defined above has dimension at least 2, then  $\sigma_2 = \sigma_1$  (where  $\sigma_1 \geq \sigma_2$  are the first two singular values of  $T$ ).