Math 307 Homework
November 20, 2015

1. Let $\mathbf{A} \in \mathrm{M}_n(\mathbb{C})$ and let $\varepsilon > 0$. Show that there is a $\mathbf{B} \in \mathrm{M}_n(\mathbb{C})$ with n distinct eigenvalues such that $\|\mathbf{A} - \mathbf{B}\| \leq \varepsilon$.

   Hint: First consider the case where $\mathbf{A}$ is upper triangular, then use the Schur decomposition.

2. (a) Prove that if $\mathbf{A} \in \mathrm{M}_n(\mathbb{C})$ is upper triangular and normal, then $\mathbf{A}$ is diagonal.

   (b) Use this fact and the Schur decomposition to prove the spectral theorem for normal matrices.

3. Show that if $D : \mathrm{M}_n(\mathbb{F}) \to \mathbb{F}$ is an alternating, multilinear function, then you can add any linear combination of the columns to any one column of a matrix A without changing the value of $D(A)$.