Math 307 Homework
November 25, 2015

1. Calculate
$$\det \begin{bmatrix} 1 & 0 & -1 & 3 \\ 2 & -3 & -2 & 5 \\ 3 & 0 & -1 & 9 \\ 2 & -3 & -2 & 6 \end{bmatrix}.$$

2. (a) Show that if $\mathbf{U} \in \mathrm{M}_n(\mathbb{C})$ is unitary, then $|\det \mathbf{U}| = 1$.

   (b) Show that if $\sigma_1, \ldots, \sigma_n$ are the singular values of $\mathbf{A} \in \mathrm{M}_n(\mathbb{C})$, then
$$|\det \mathbf{A}| = \sigma_1 \cdots \sigma_n.$$

3. Suppose that $\mathbf{A} \in \mathrm{M}_{m+n}(\mathbb{F})$ has the form
$$\mathbf{A} = \begin{bmatrix} \mathbf{B} & \mathbf{C} \\ \mathbf{0} & \mathbf{D} \end{bmatrix}$$
for some $\mathbf{B} \in \mathrm{M}_m(\mathbb{F})$, $\mathbf{D} \in \mathrm{M}_n(\mathbb{F})$, and $\mathbf{C} \in \mathrm{M}_{m,n}(\mathbb{F})$. Show that $\det \mathbf{A} = \det \mathbf{B} \det \mathbf{D}$.

   Hint: Prove this for fixed m by induction on n.