

Math 307 Homework
November 30, 2015

1. Let $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.
 - (a) Show that \mathbf{A} and \mathbf{I}_2 have the same trace, determinant, and characteristic polynomial.
 - (b) Show that \mathbf{A} and \mathbf{I}_2 are *not* similar.
2. Let $\mathbf{A} \in M_n(\mathbb{F})$, and let $p \in \mathcal{P}(\mathbb{F})$ be any polynomial. Prove that $p(\mathbf{A})$ lies in the span of

$$(\mathbf{I}_n, \mathbf{A}, \mathbf{A}^2, \dots, \mathbf{A}^{n-1}).$$

Warning: Make sure you don't fall into the trap of using the same letter n to stand for more than one thing. The degree of the polynomial p might be much larger than n .