Math 307 Homework
August 18, 2015

1. Give examples of linear systems of each of the following types, if possible. Explain how you know they have the claimed properties, or else explain why there is no such system.

   (a) Underdetermined and inconsistent.

   (b) Undetermined with a unique solution.

   (c) Overdetermined with more than one solution.

2. (a) Explain why every linear system (over $\mathbb{R}$) has either zero solutions, one unique solution, or infinitely many solutions.

   (b) Why doesn't this contradict the fact that the system of equations

   $$x^2 - y = 1$$
   $$x + 2y = 3$$

   has exactly two solutions?

3. Under what conditions is the vector $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$ in

   $$U := \left\langle \begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \\ 0 \end{bmatrix} \right\rangle ?$$

   Give your answer in terms of an equation satisfied by the entries b_1, b_2, b_3, b_4.

4. Which of the following vectors are in the set U from the previous problem?

   (a) $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ (b) $\begin{bmatrix} -1 \\ -1 \\ -2 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} -4 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} -3 \\ 2 \\ -1 \\ 1 \end{bmatrix}$