Math 307 Homework
August 30, 2015

1. Prove that any two non-zero vectors in $\mathbb{R}^2$ which are not collinear span $\mathbb{R}^2$.
   Hint: The hard part here is figuring out how to express and then use the fact that the vectors are not collinear.

2. Let $\mathbb{F} = \{a + b\sqrt{5} : a, b \in \mathbb{Q}\}$. Show that $\mathbb{F}$ is a field.

   Hint: Since $\mathbb{F} \subseteq \mathbb{R}$, you can take things like associativity, commutativity, and the distributive law as known. What you need to check is that $0, 1 \in \mathbb{F}$, that the sum and product of two numbers in $\mathbb{F}$ is actually in $\mathbb{F}$, and that the additive and multiplicative inverses of a number in $\mathbb{F}$ are in $\mathbb{F}$.

3. (a) Prove part 3 of Theorem 1.5.
   (b) Prove part 8 of Theorem 1.5.