Math 307 Homework
September 2, 2015

1. Determine which of the following are and are not subspaces of the given vector space, and justify your answers.
   (a) The x axis in $\mathbb{R}^3$.
   (b) The set $\left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| x, y \geq 0 \right\}$ in $\mathbb{R}^2$ (i.e., the first quadrant of the plane).
   (c) The set $\left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| x, y \geq 0 \right\} \cup \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| x, y \leq 0 \right\}$ in $\mathbb{R}^2$ (i.e., the first and third quadrants of the plane).
   (d) The set of solutions of the linear system
   $$\begin{aligned} x - y + 2z &= 4 \\ 2x - 5z &= -1. \end{aligned}$$

2. (a) Show that $\mathbb{C}$ is a vector space over $\mathbb{R}$.
   (b) Show that $\mathbb{Q}$ is not a vector space over $\mathbb{R}$.

3. The **trace** of an $n \times n$ matrix $\mathbf{A} = [a_{ij}]_{\substack{1 \leq i \leq n \\ 1 \leq j \leq n}}$ is
   $$\operatorname{tr} \mathbf{A} = \sum_{i=1}^{n} a_{ii}.$$
   Show that $S = \{\mathbf{A} \in \mathrm{M}_n(\mathbb{F}) \mid \operatorname{tr} \mathbf{A} = 0\}$ is a subspace of $\mathrm{M}_n(\mathbb{F})$.

4. Let V be a vector space, and suppose that U and W are both subspaces of V. Show that
   $$U \cap W := \{v | v \in U \text{ and } v \in W\}$$
   is a subspace of V.