Math 307 Homework
September 4, 2015

1. Let $C(\mathbb{R})$ be the vector space (over $\mathbb{R}$) of continous functions $f : \mathbb{R} \to \mathbb{R}$. Define $T : C(\mathbb{R}) \to C(\mathbb{R})$ by
$$[Tf](x) = f(x)\cos(x).$$
Show that T is a linear map.

2. Give an explicit isomorphism between the set of solutions of the linear system (over $\mathbb{R}$)
$$\begin{aligned} w - x + 0y + 3z &= 0, \\ w - x + y + 5z &= 0, \\ 2w - 2x - y + 4z &= 0 \end{aligned}$$
and $\mathbb{R}^2$.

3. Define $\boldsymbol{T} : \mathbb{R}^3 \to \mathbb{R}^3$ by
$$\boldsymbol{T}\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} y \\ z \\ 0 \end{bmatrix}.$$
   (a) Is $\boldsymbol{T}$ linear?
   (b) Is $\boldsymbol{T}$ injective?
   (c) Is $\boldsymbol{T}$ surjective?

   Justify all your answers.

4. If $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, the **line segment** between $\mathbf{x}$ and $\mathbf{y}$ is the set
$$L := \{(1-t)\mathbf{x} + t\mathbf{y} | 0 \le t \le 1\}.$$
Show that if $\boldsymbol{T} : \mathbb{R}^2 \to \mathbb{R}^2$ is linear, then $\boldsymbol{T}(L)$ is also a line segment.

   Remark: This is one way in which linear maps really do have something to do with lines.