1. Show that $\mathbb{E}[X]^2 \leq \mathbb{E}[X^2]$ with equality if and only if $X = \mathbb{E}[X]$ with probability 1.

2. Suppose you perform a sequence of independent trials of an experiment. Let A be some set of possible outcomes, and let X_i be the indicator that the ith trial has outcome in A. What does the strong law of large numbers tell you about this situation? What does this have to do with the frequentist definition of probability proposed at the beginning of the term?

3. Let X be a random variable with moment generating function $M(t)$. Prove that

$$\mathbb{P}[X \geq a] \leq e^{-ta} M(t) \qquad \text{for } t > 0$$

and

$$\mathbb{P}[X \leq a] \leq e^{-ta} M(t) \qquad \text{for } t < 0.$$