Math 423 Extra problem 2

Let μ be a measure on $\mathcal{B}_{\mathbb{R}}$ such that

1. $\mu(E + x) = \mu(E)$ for all $x \in \mathbb{R}$ and all $E \in \mathcal{B}_{\mathbb{R}}$;

2. for some bounded interval I, $\mu(I) < \infty$.

Prove that there is an $a > 0$ such that $\mu = am$, where m denotes Lebesgue measure on $\mathbb{R}$. That is, up to rescaling, Lebesgue measure is the unique translation-invariant Borel measure on $\mathbb{R}$.