

Let X be a non-compact locally compact Hausdorff space and let $X^* = X \cup \{\infty\}$. If

$$\mathcal{T} = \{U \subseteq X : U \text{ open}\} \cup \{V \subseteq X^* : \infty \in V, V^c \text{ compact in } X\},$$

show that \mathcal{T} is a topology on X^* .