Suppose that $X$ and $Y$ are compact topological spaces. Show, without using Alexander's Lemma, that $X \times Y$ is compact. (You may still use the fact that sets of the form $U \times V$, where $U$ is open in $X$ and $V$ is open in $Y$, form a base for the product topology.)

