1. Let $B(t)$ be a standard Brownian motion. Show that the following are Ornstein–Uhlenbeck processes:

$$U_1(t) = e^{-\beta t} B(e^{2\beta t} - 1)$$

$$U_3(t) = B(t) - \beta \int_0^t e^{-\beta(t-s)} B(s)ds.$$

2. Let $X_{1,t}, \ldots, X_{n,t}$ be independent standard Brownian motions, and let

$$Y_t := \sum_{j=1}^{n} X_{j,t}^2.$$

Show that Y_t is a diffusion and find the drift and instantaneous variance.