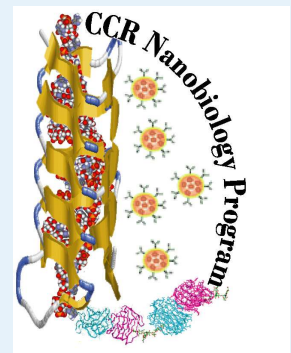
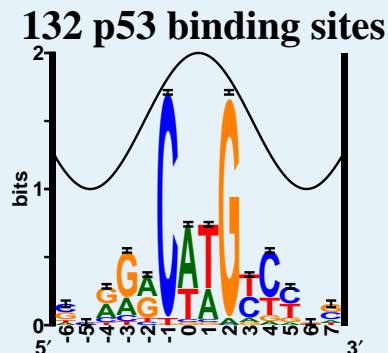




Efficiency of Molecular Machines

Thomas D. Schneider, Ph.D.

National Cancer Institute at Frederick
Center for Cancer Research Nanobiology Program
Molecular Information Theory Group



El Duomo, Florence, Italy



Information Theory: One-Minute Lesson

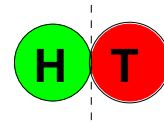
number of symbols	number of bits	example
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M

B

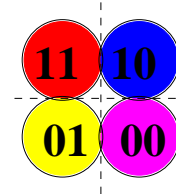
2

1



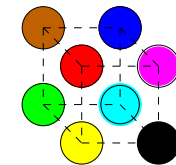
4

2



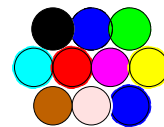
8

3



$$M=2^B$$

$$B=\log_2 M$$



Information Theory: One-Minute Lesson

number of
symbols

number of
bits

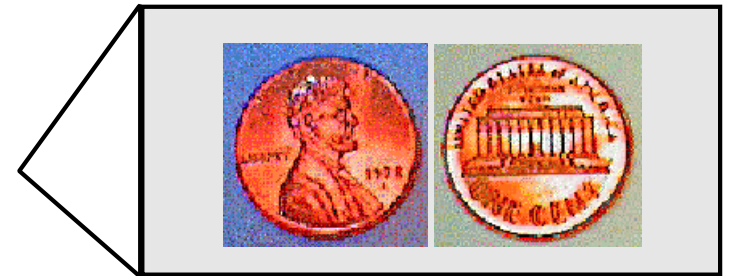
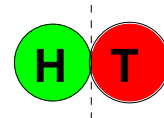
example

M

B

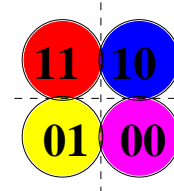
2

1



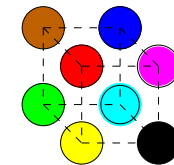
4

2



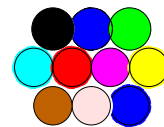
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Information Theory: One-Minute Lesson

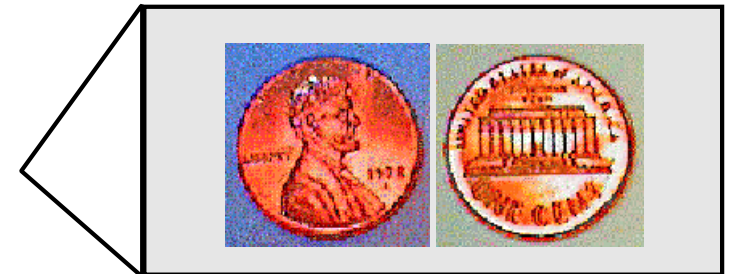
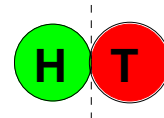
number of symbols	number of bits	example
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M

B

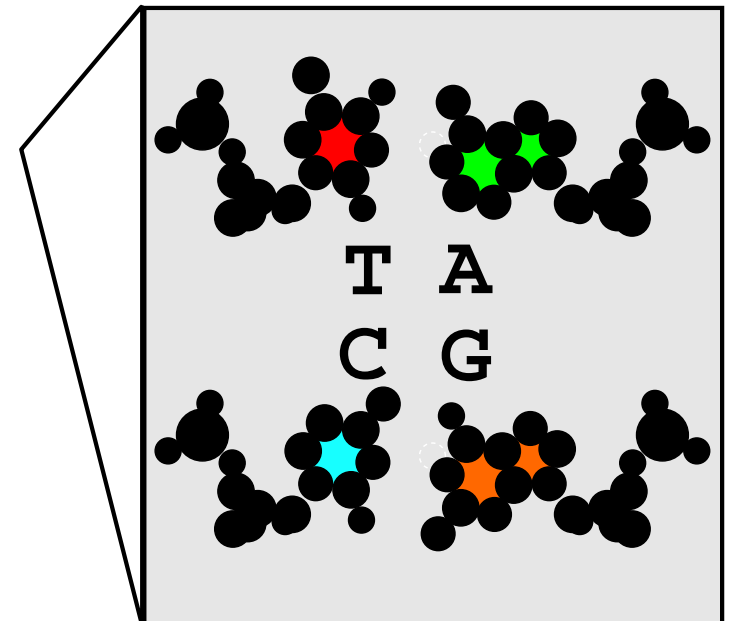
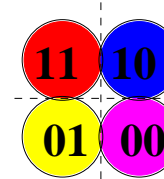
2

1



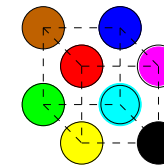
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2



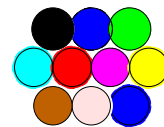
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Information Theory: One-Minute Lesson

number of
symbols

number of
bits

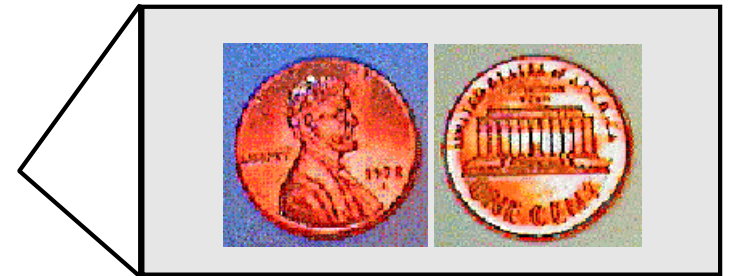
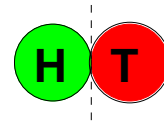
example

M

B

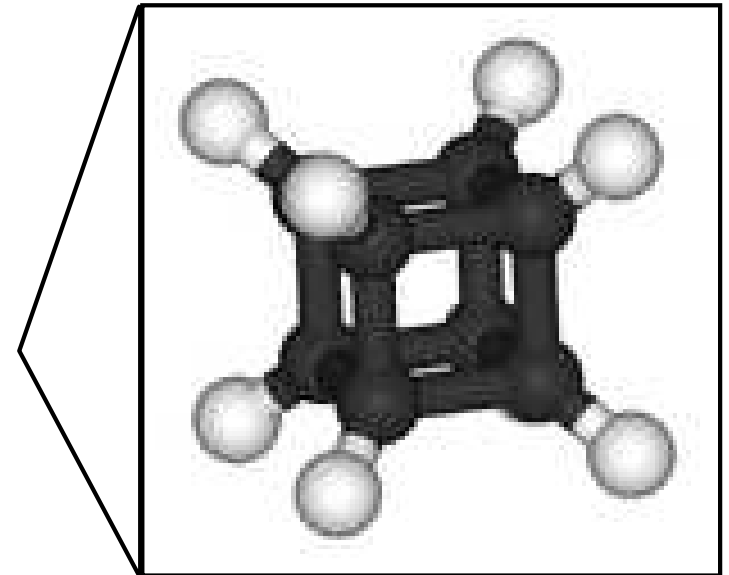
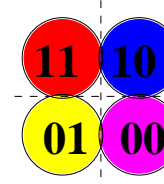
2

1



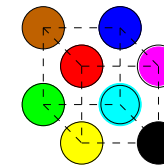
4

2



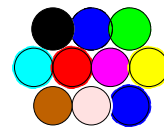
8

3



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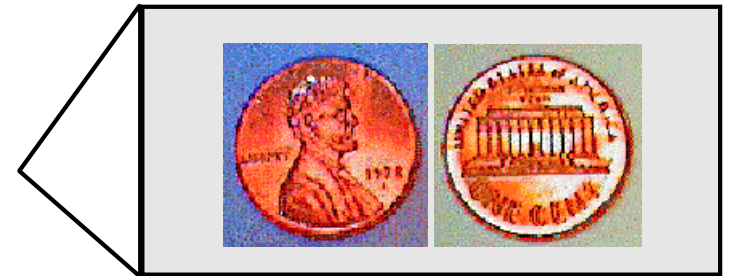
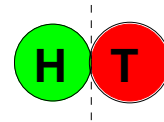
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B

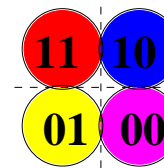
2

1



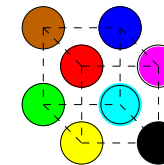
4

2



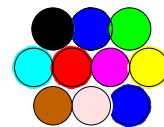
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3

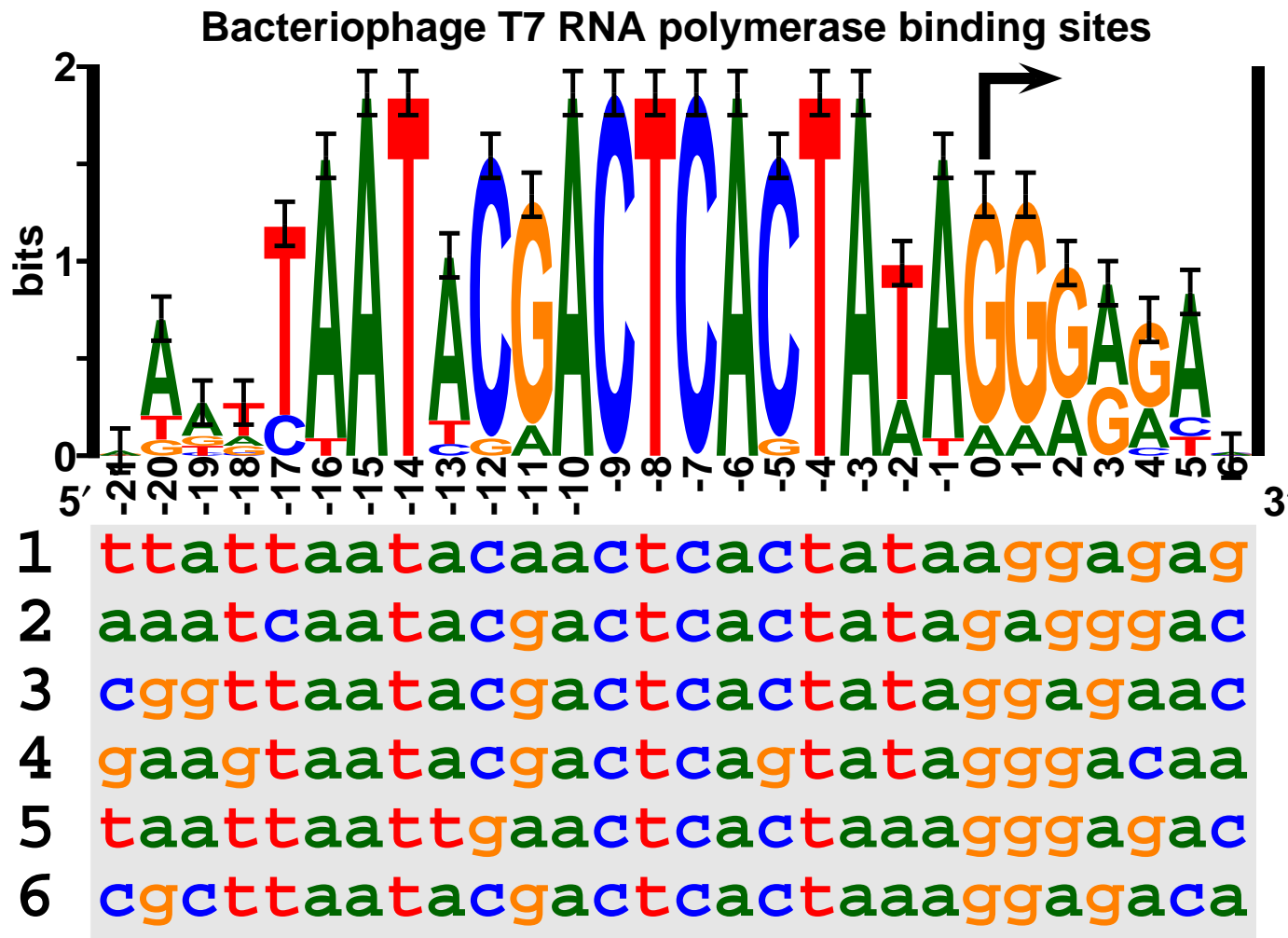


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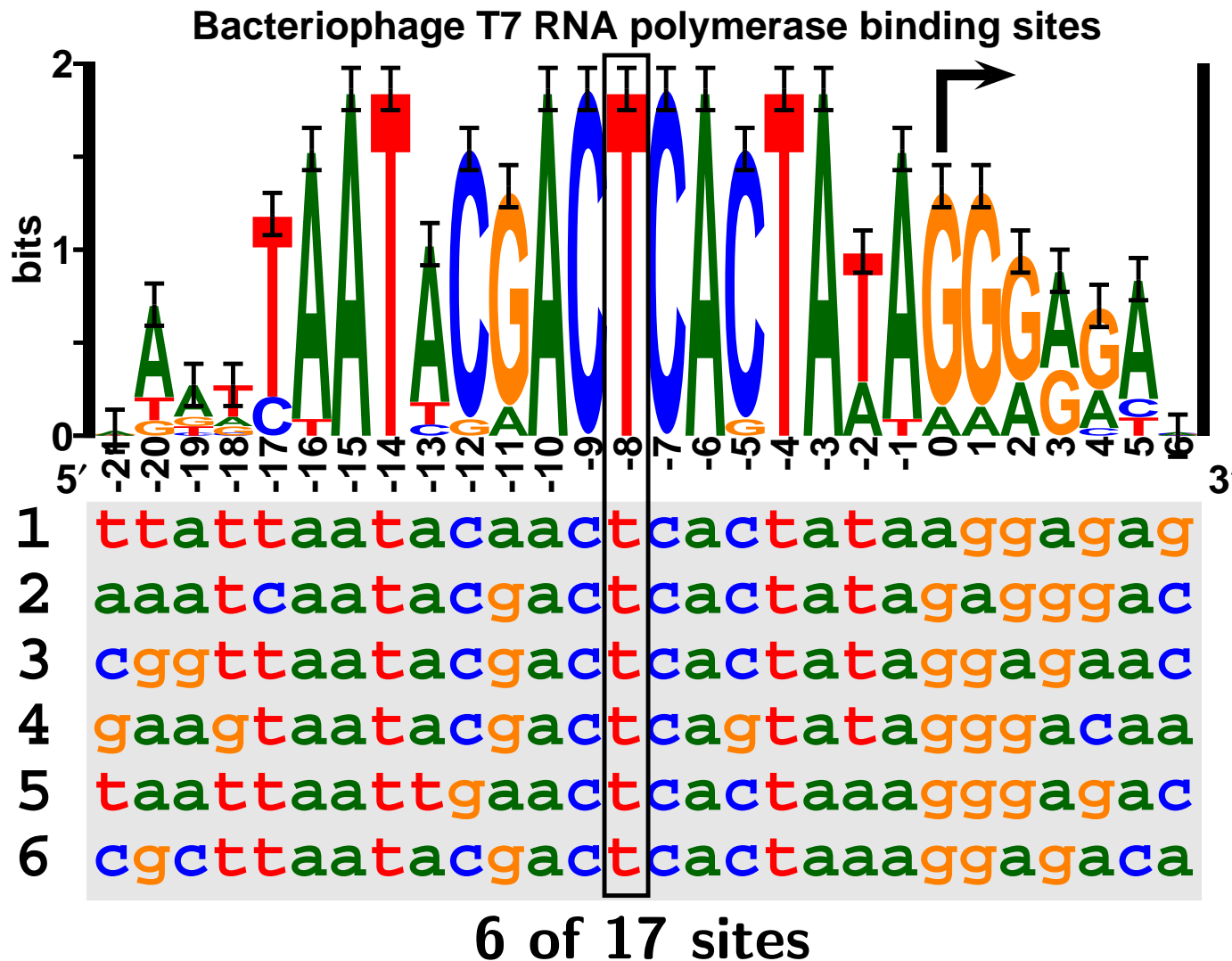
Sequence Logo



Schneider &
Stephens
Nucl. Acids Res.
18: 6097-6100
1990

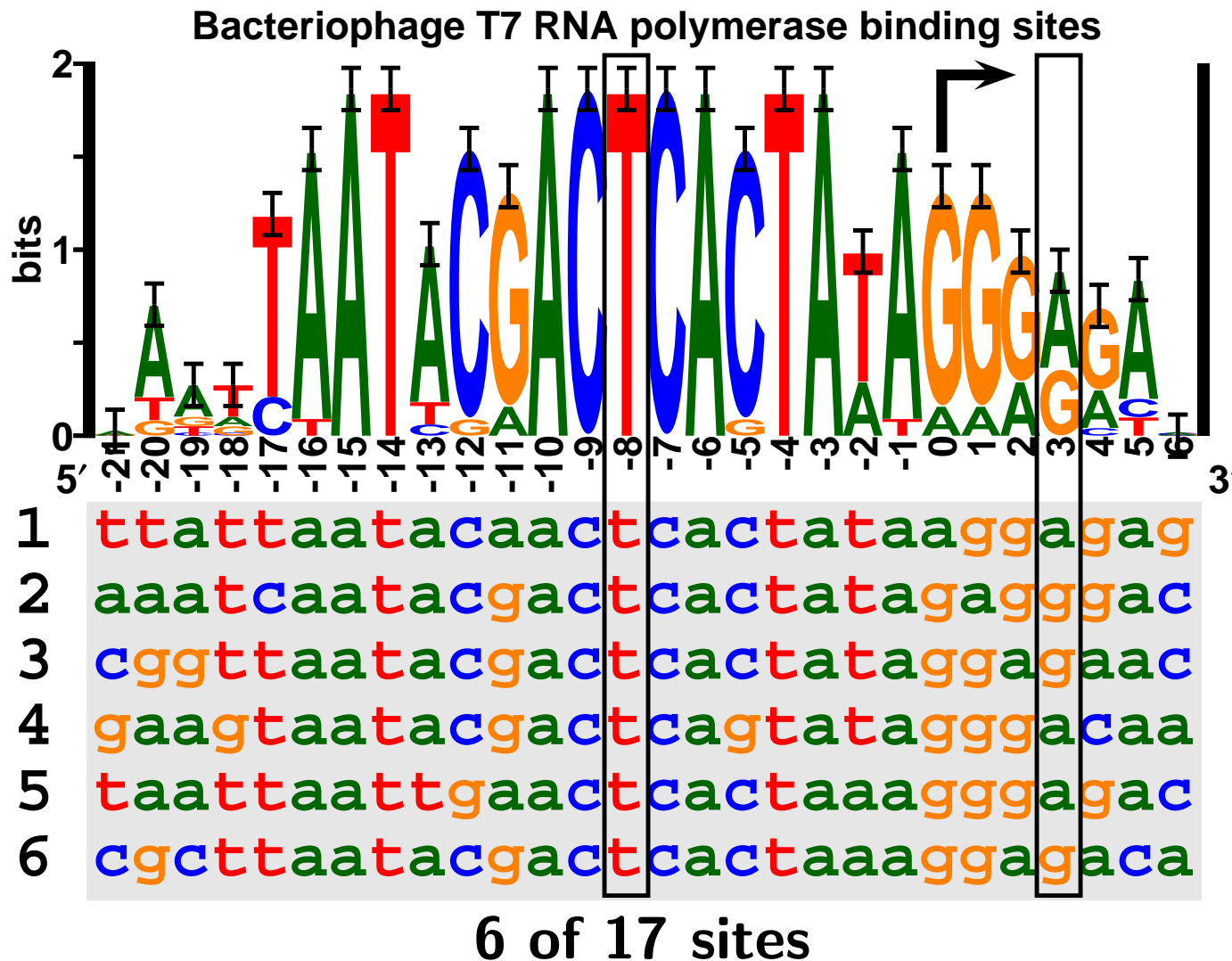
6 of 17 sites

Sequence Logo



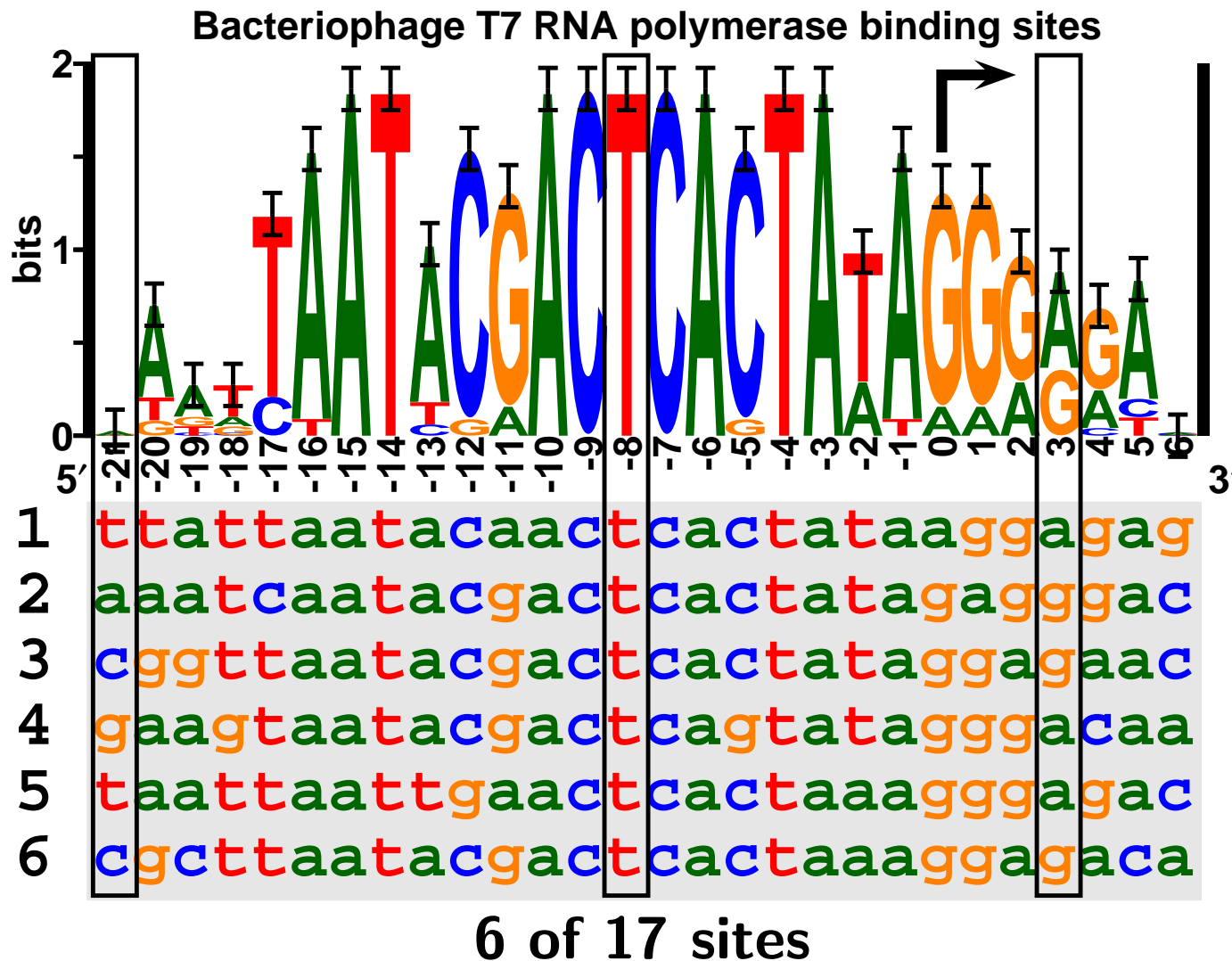
Schneider &
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Nucl. Acids Res.
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Sequence Logo



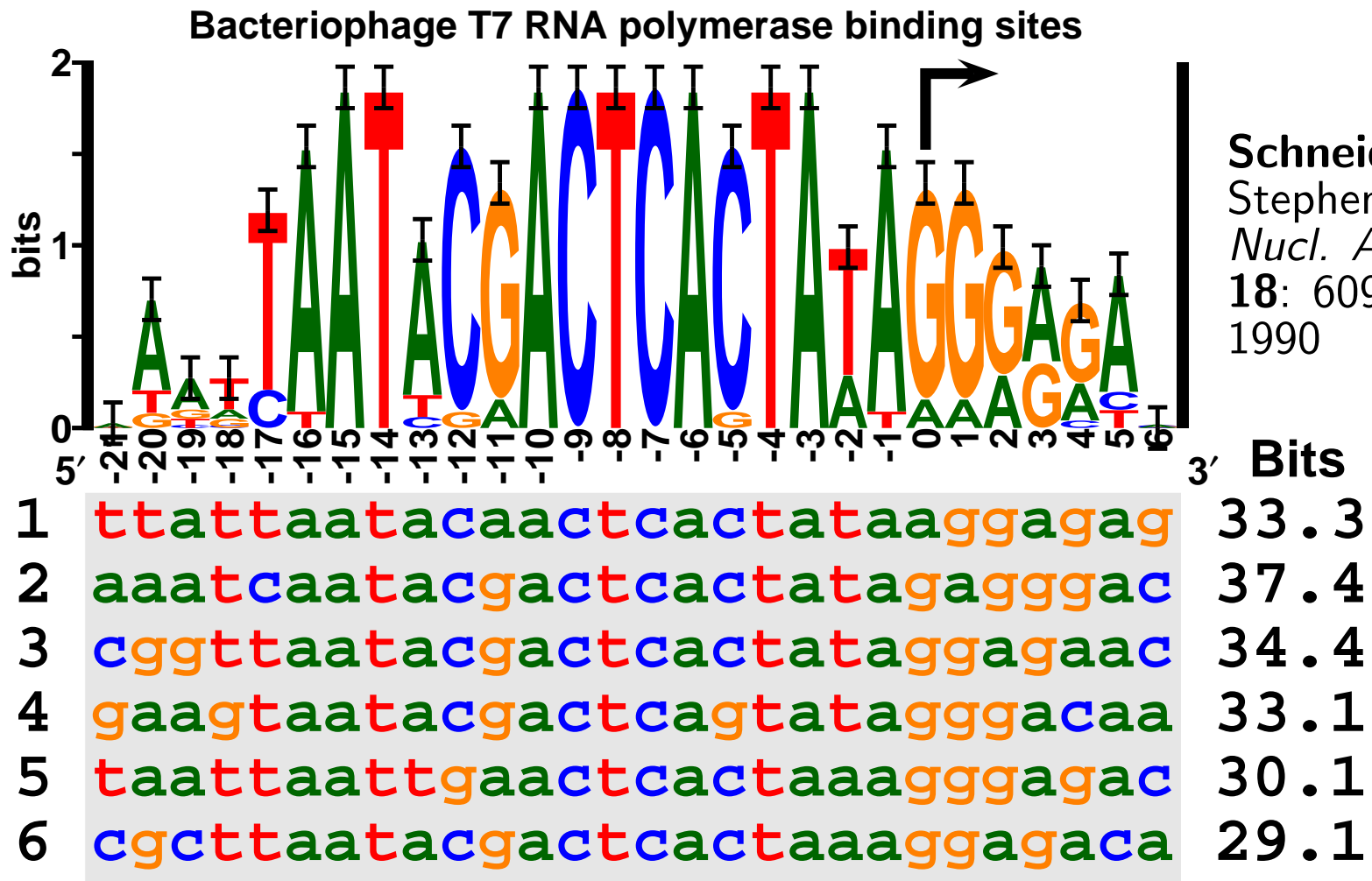
Schneider &
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Sequence Logo



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1990

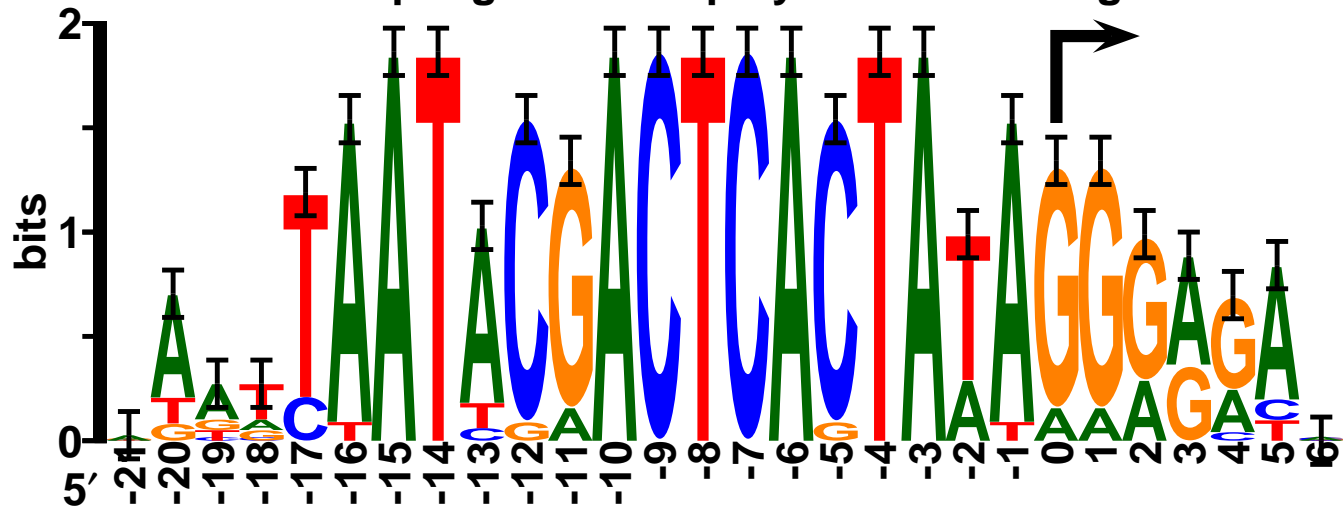
Sequence Logo and Sequence Walker



Schneider &
Stephens
Nucl. Acids Res.
18: 6097-6100
1990

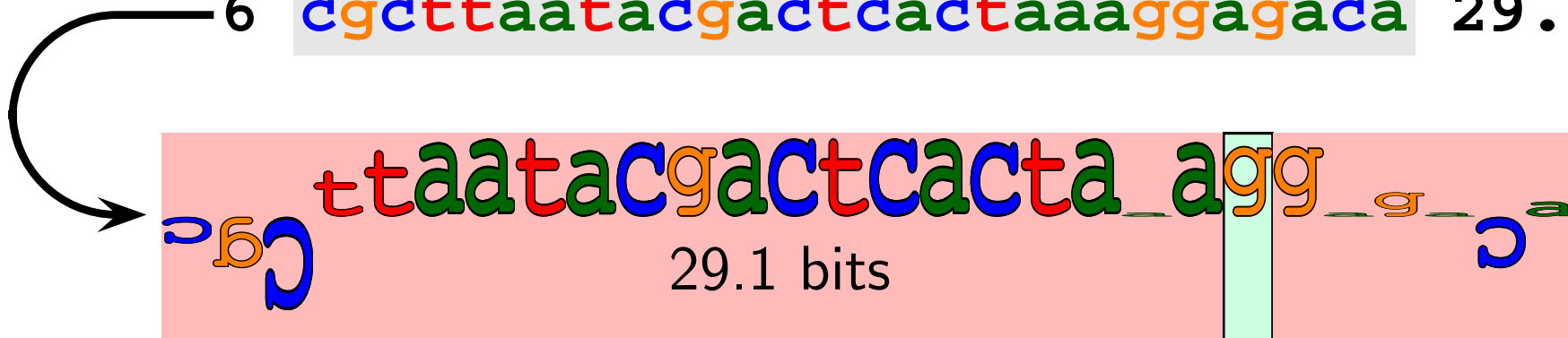
Sequence Logo and Sequence Walker

Bacteriophage T7 RNA polymerase binding sites



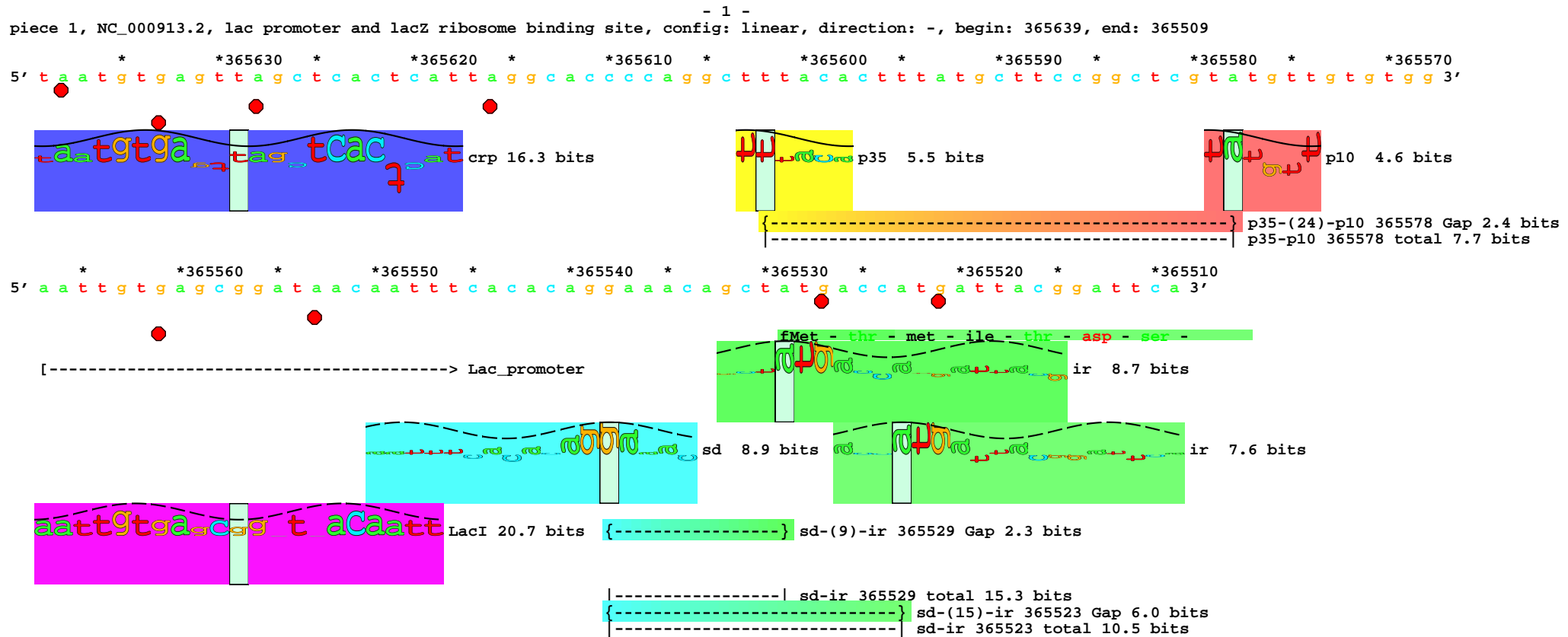
Schneider &
Stephens
Nucl. Acids Res.
18: 6097-6100
1990

		Bits
1	ttattaatacaactcactataaggagag	33.3
2	aatcaatacgactcactatagagggac	37.4
3	cggttaatacgactcactataggagaac	34.4
4	gaagtaatacgactcagtatagggaaca	33.1
5	taattaattgaactcactaaaggggagac	30.1
6	cgcttaatacgactcactaaaggagaca	29.1

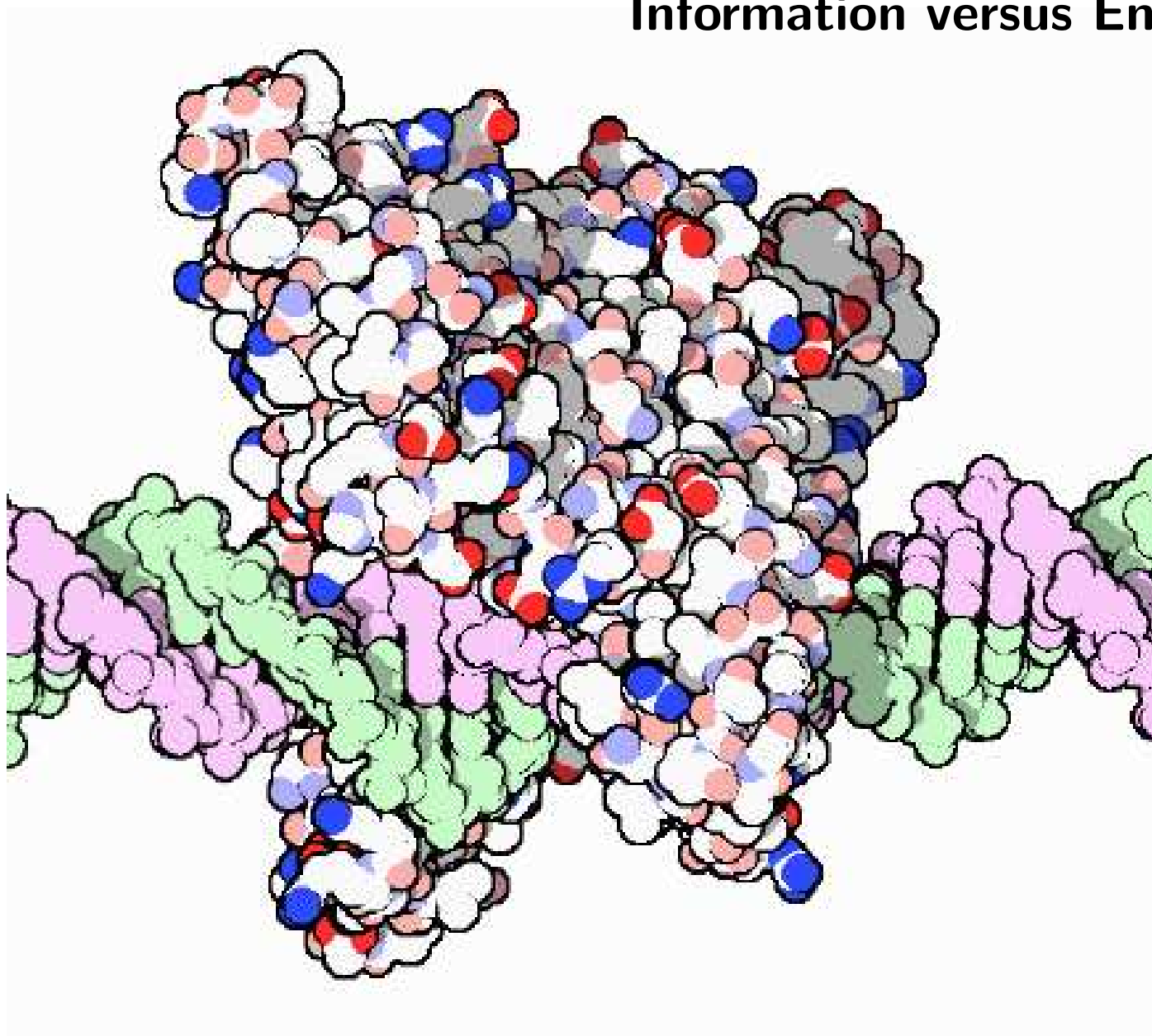


Sequence
Walker
Patent
5,867,402

Sequence Walkers in the Lac Promoter

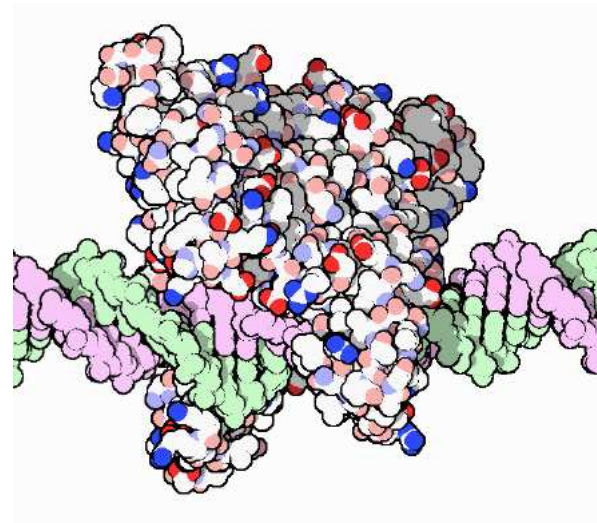


Information versus Energy



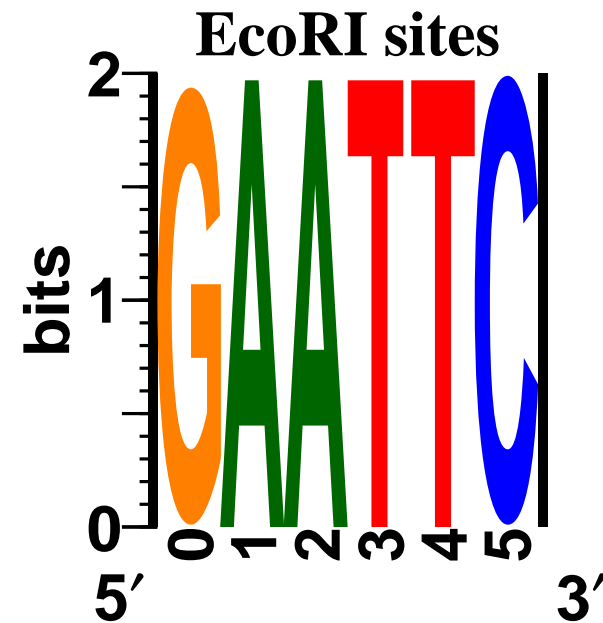
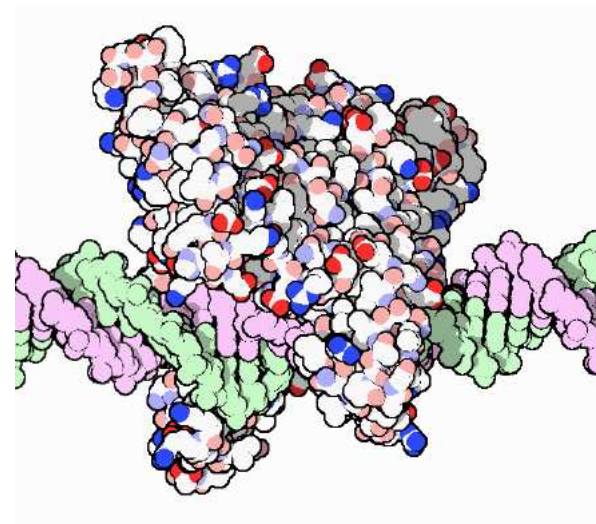
Information of EcoRI DNA Binding

- EcoRI - restriction enzyme



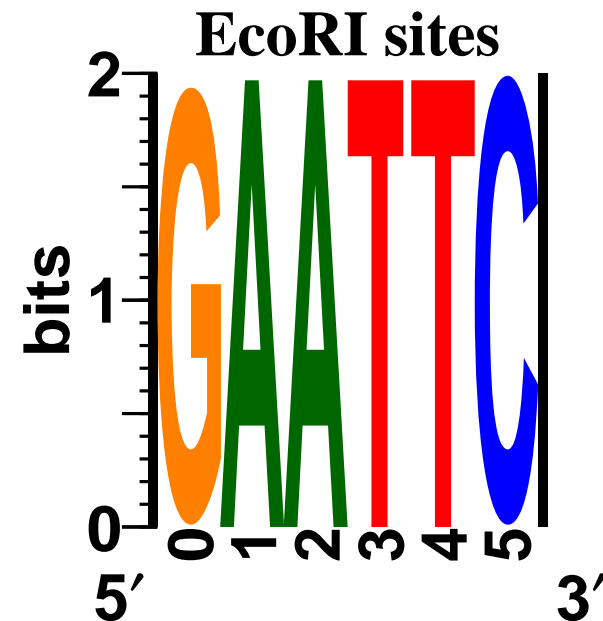
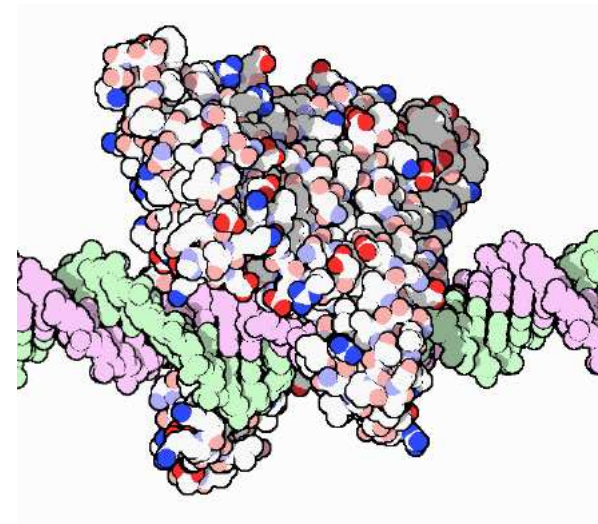
Information of EcoRI DNA Binding

- EcoRI - restriction enzyme
- EcoRI binds DNA at 5' GAATTC 3'



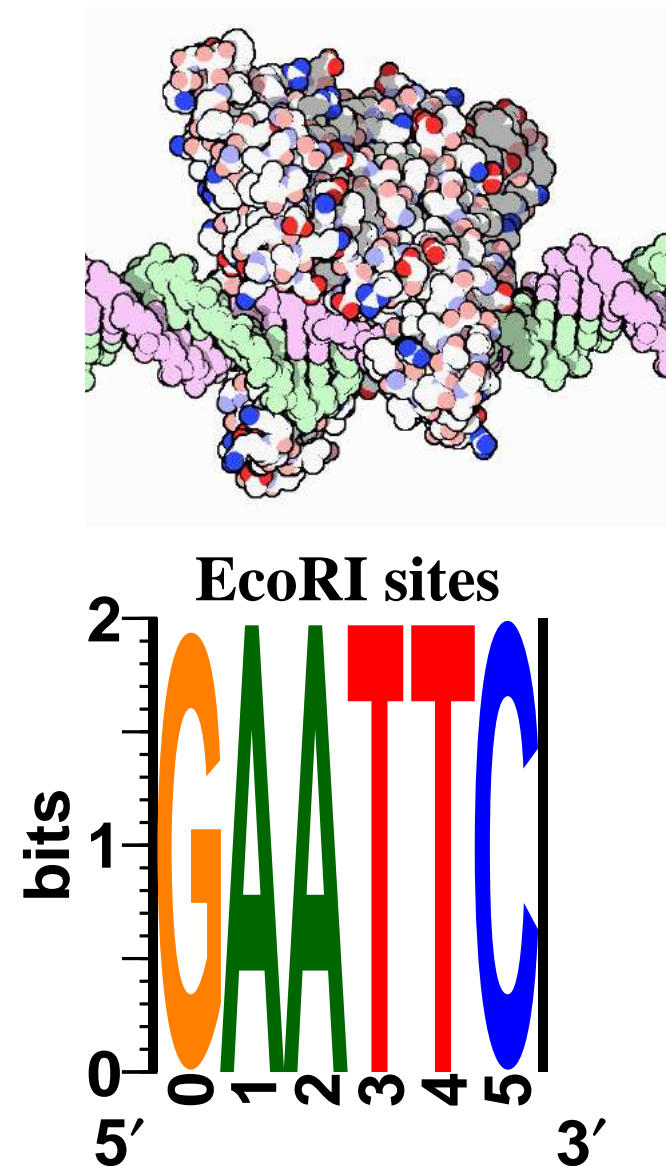
Information of EcoRI DNA Binding

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- $4^6 = 4096$ possible DNA hexamers



Information of EcoRI DNA Binding

- EcoRI - restriction enzyme
- EcoRI binds DNA at 5' GAATTC 3'
- $4^6 = 4096$ possible DNA hexamers
- information required:
 $\log_2 4096 = 12$ bits
or
 $6 \text{ bases} \times 2 \text{ bits per base} = \boxed{12 \text{ bits}}$



Energy Dissipation by EcoRI

- Measured specific binding constant:

$$K_{spec} = 1.6 \times 10^5$$

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$$\mathcal{E}_{min} = k_B T \ln 2 \quad (\text{joules per bit})$$

- Number of bits that could have been selected:

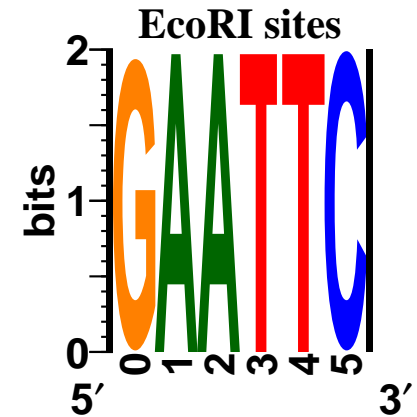
$$\begin{aligned} R_{energy} &= -\Delta G^{\circ} / \mathcal{E}_{min} \\ &= k_B T \ln K_{spec} / k_B T \ln 2 \\ &= \log_2 K_{spec} && \Leftarrow \text{SO SIMPLE!} \\ &= \boxed{17.3 \text{ bits per binding}} \end{aligned}$$

Information/Energy = Efficiency of EcoRI

EcoRI could have made 17.3 binary choices

Information/Energy = Efficiency of EcoRI

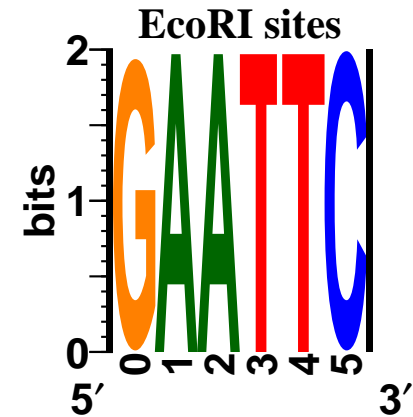
EcoRI could have made 17.3 binary choices
...but it only made 12 choices.



Information/Energy = Efficiency of EcoRI

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Efficiency is
'WORK' DONE / ENERGY DISSIPATED

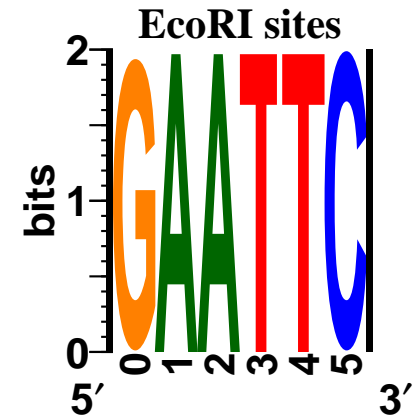


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$$\frac{12 \text{ bits per binding}}{17.3 \text{ bits per binding}} = 0.7$$



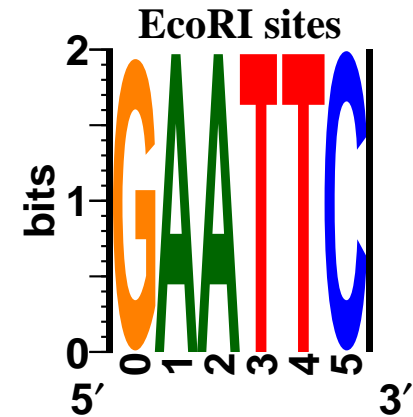
Information/Energy = Efficiency of EcoRI = 70%

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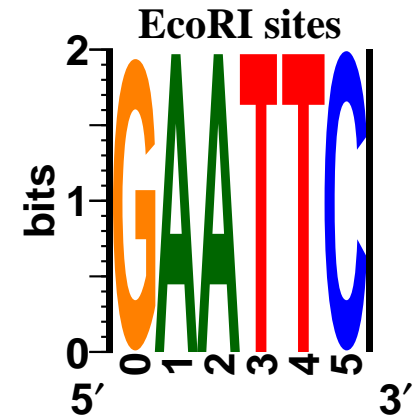
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18 out of 19 DNA binding proteins give ~70% efficiency.

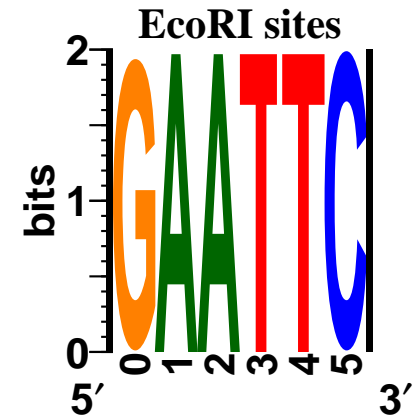


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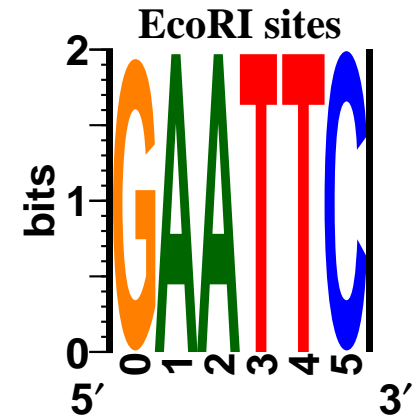
**70% efficiency also appears widely in biology:
rhodopsin, muscle and other systems.**

Information/Energy = Efficiency of EcoRI = 70%

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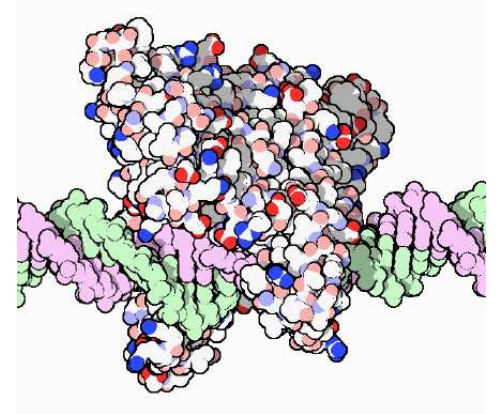
**70% efficiency also appears widely in biology:
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Why 70% efficiency?

Theoretical Efficiency

- For molecular states of molecules with d_{space} 'parts' P_y energy is dissipated for noise N_y and

$$C = d_{space} \log_2(P_y/N_y + 1) \leftarrow \text{machine capacity}$$

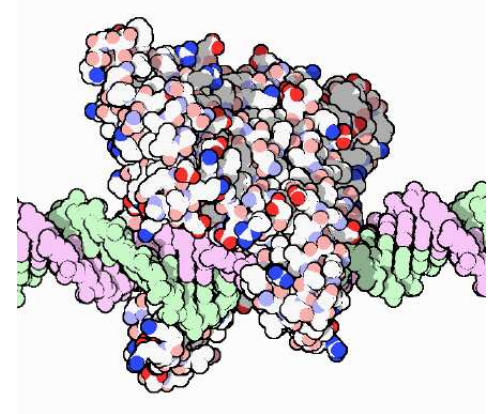


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$$\epsilon_t \leq \frac{\ln\left(\frac{P_y}{N_y} + 1\right)}{\frac{P_y}{N_y}} \leftarrow \text{molecular efficiency}$$

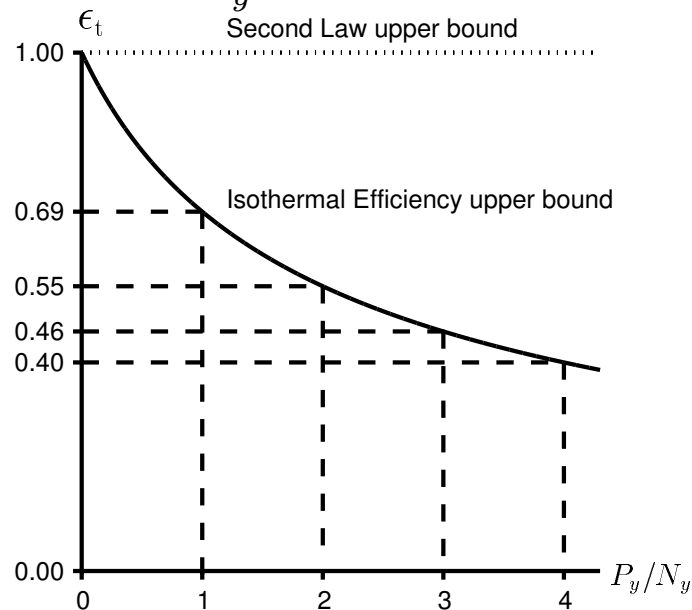


Theoretical Efficiency

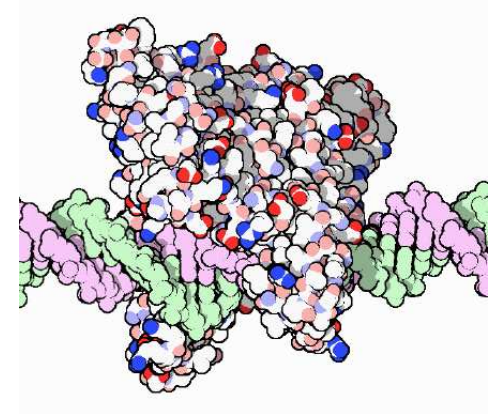
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The curve is an upper bound

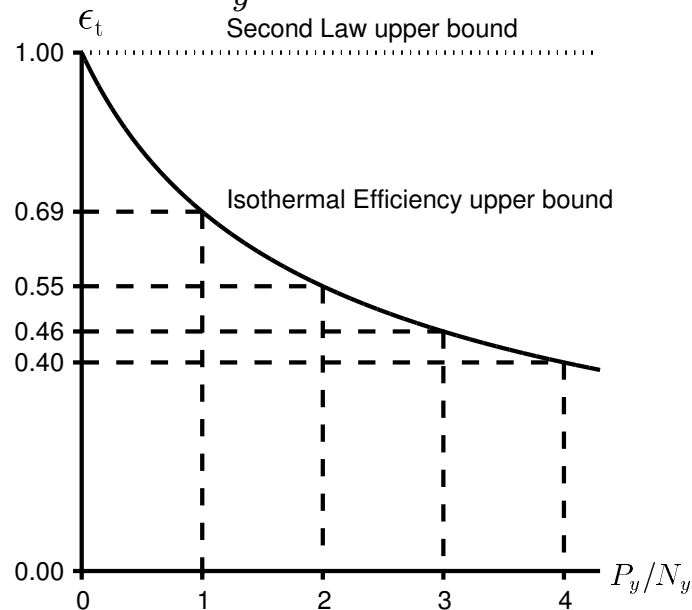


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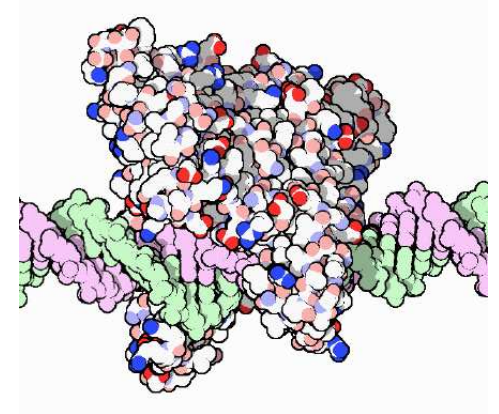
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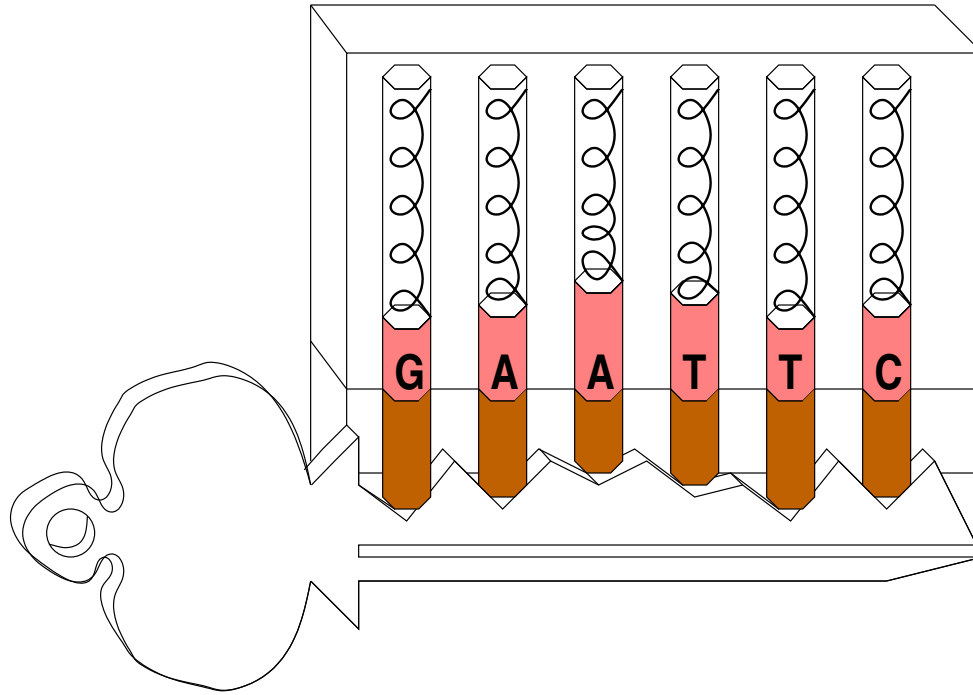


The curve is an upper bound

- If $P_y/N_y = 1$ the efficiency is 70%!

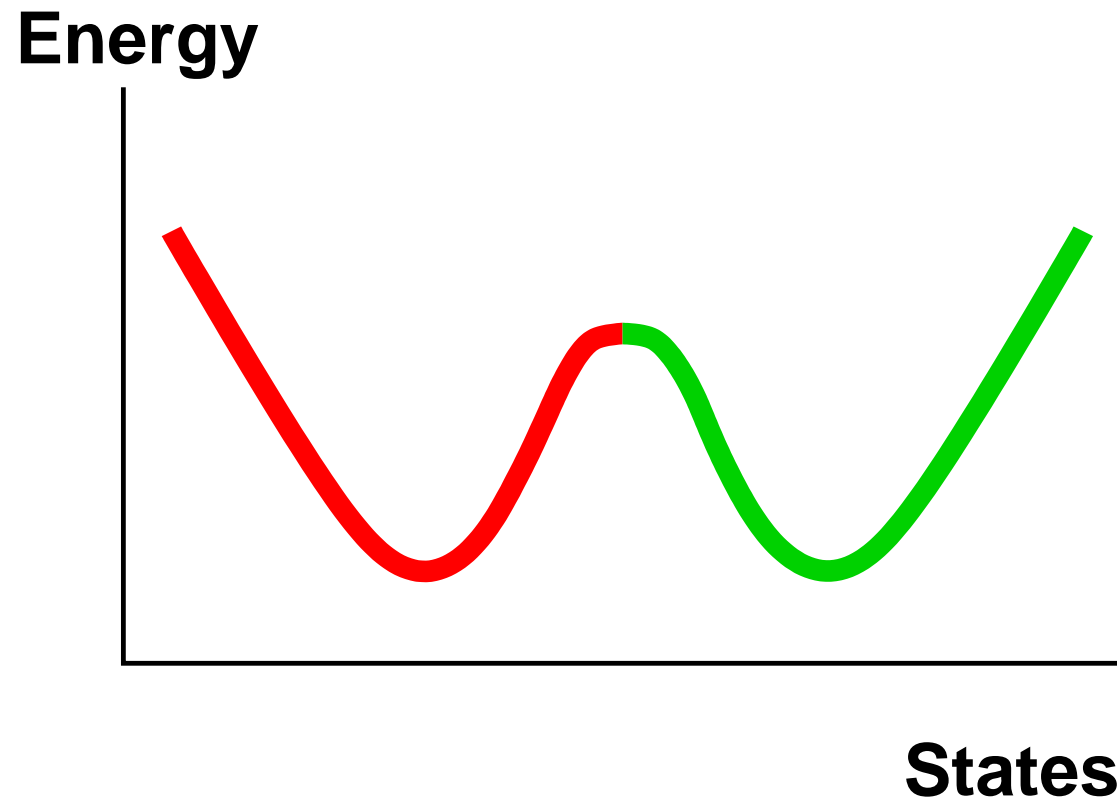


Lock and Key



**Like a key in a lock
which has many independent pins,
it takes many numbers
to describe the vibrational state
of a molecular machine**

1 Dimension



1 dimension is too simple!

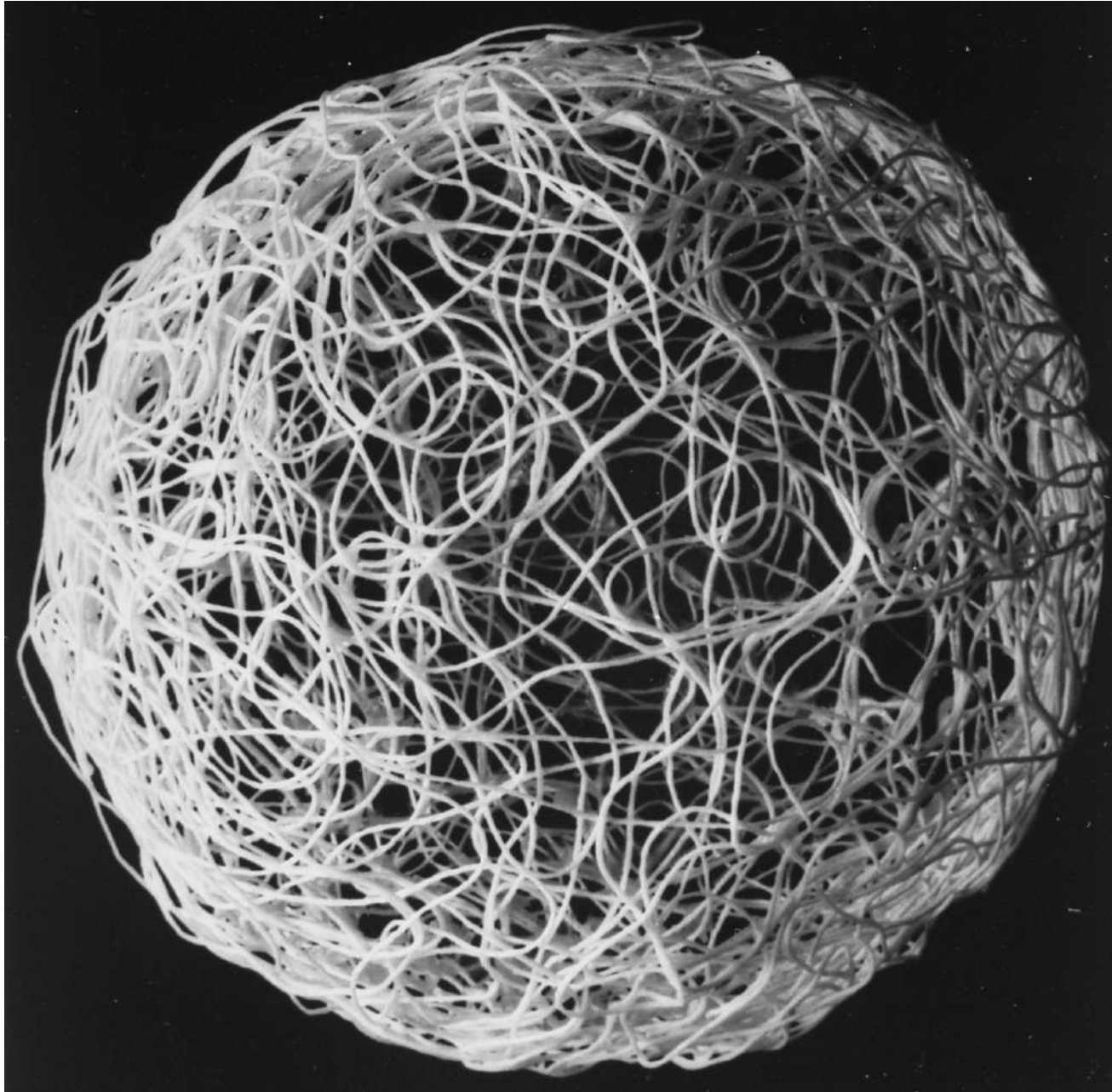
Bowls in 2 Dimensions



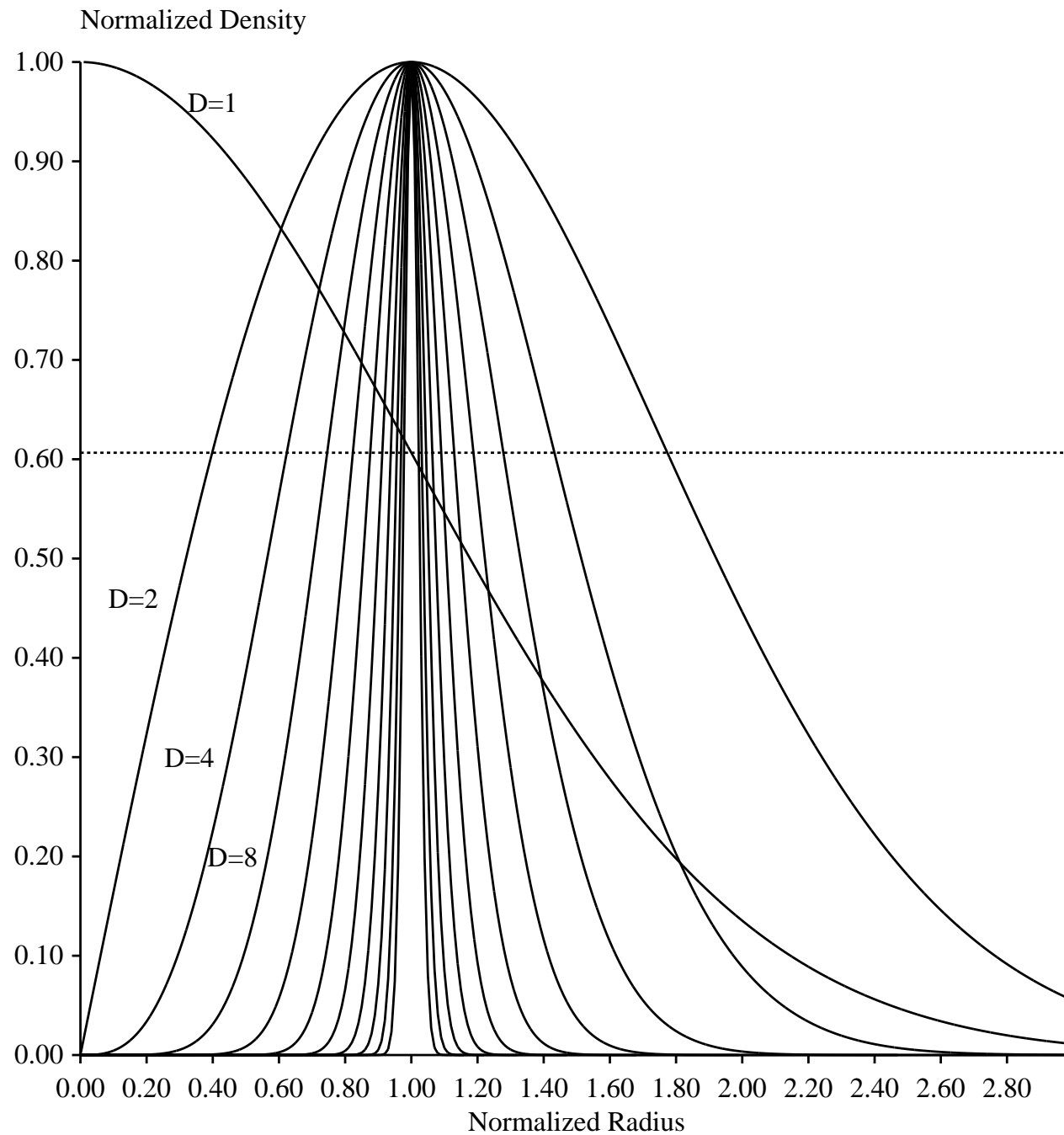
Spheres in 3 Dimensions



N Dimensional Sphere



Spheres tighten in high dimensions



$$\text{Energy} = \frac{1}{2} \text{Mass} \times \text{velocity}^2$$

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Energy in the molecule = Noise = N

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$$\text{maximum velocity} \propto \sqrt{N}$$

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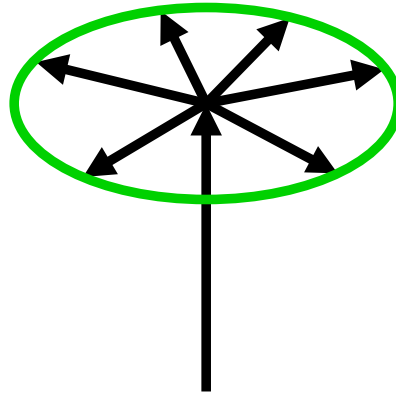
$$\text{Energy in the molecule} = \text{Noise} = N$$

$$\text{maximum velocity} \propto \sqrt{N}$$

$$\text{sphere radius} \propto \sqrt{N}$$

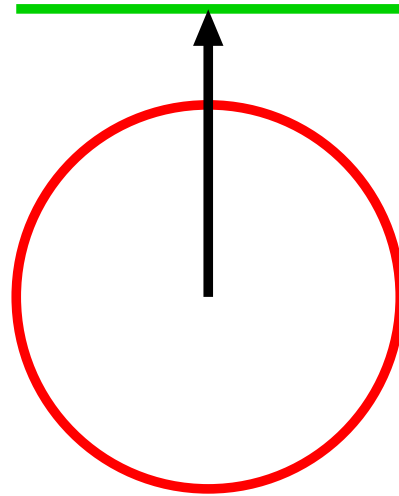
Sphere Packing



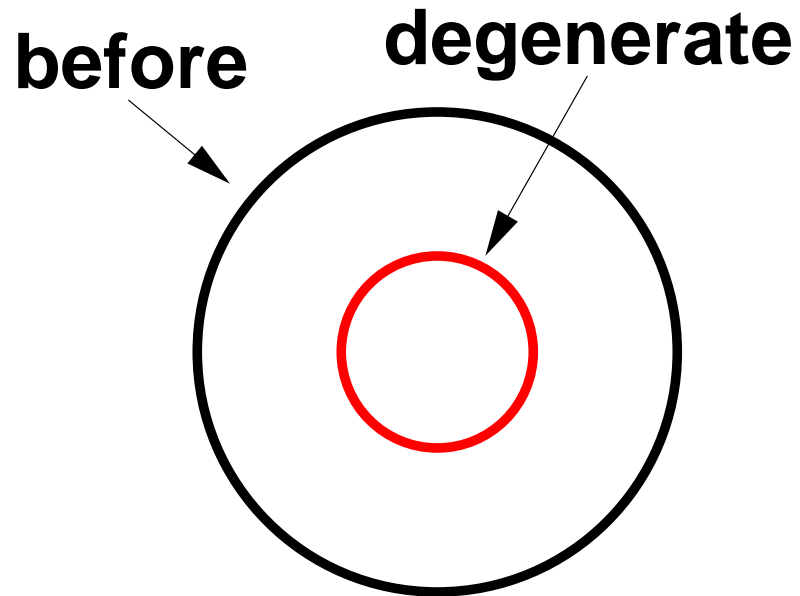


**In 100 dimensions
99% of the thermal noise
is at right angles
to a given direction!**

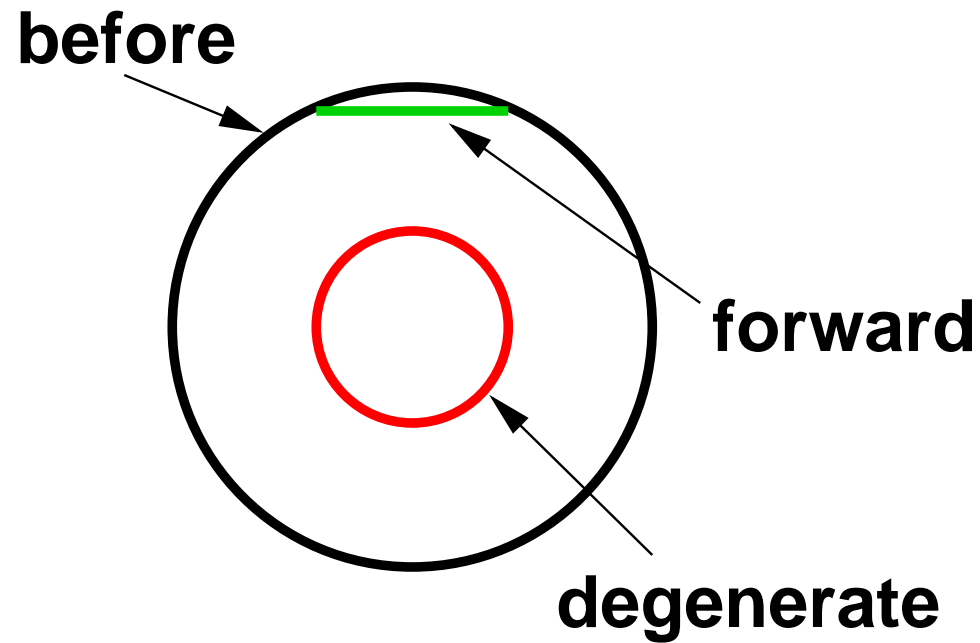
two



**Two spheres in
high dimensional space**



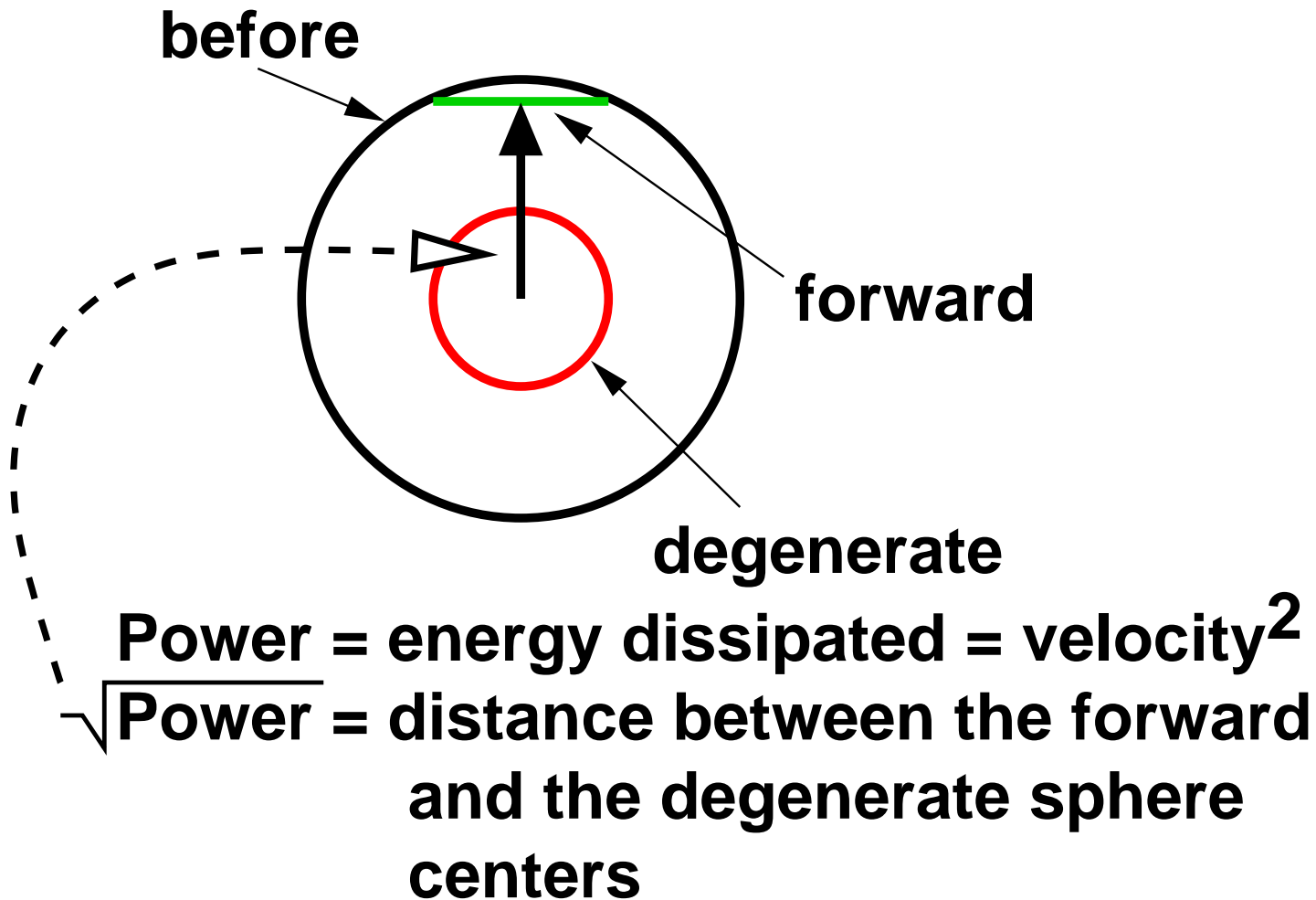
**Hypothesis:
there is a sphere
in the middle
of the before sphere**

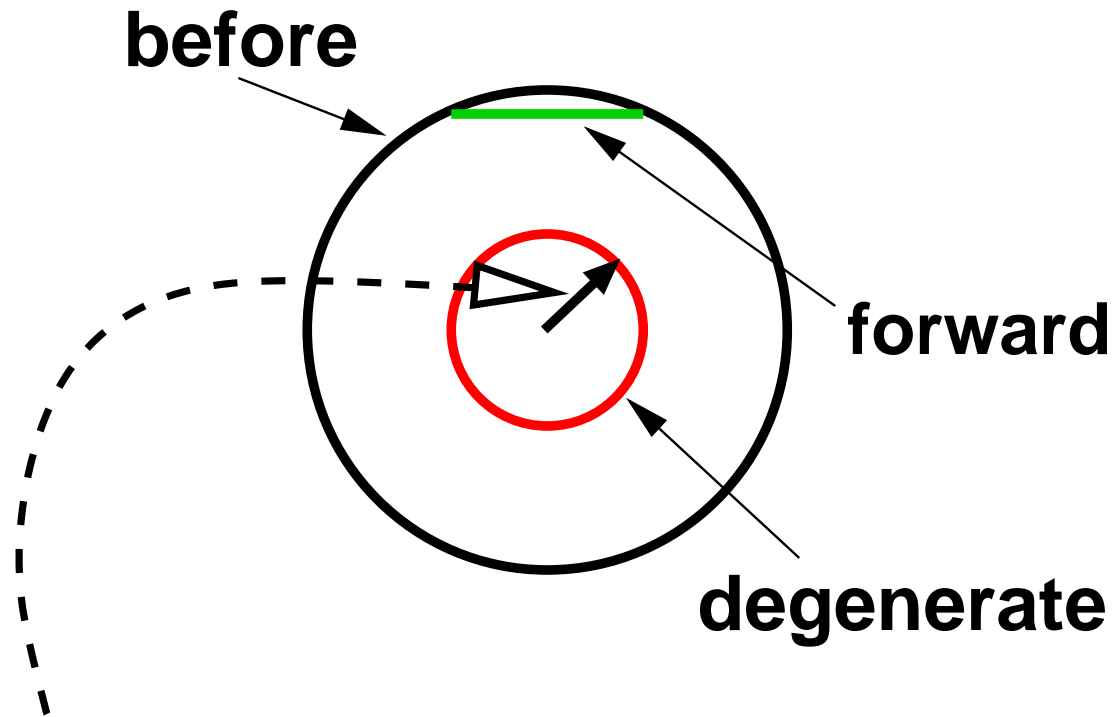


To do useful selections
the molecular machine

must avoid the degenerate sphere

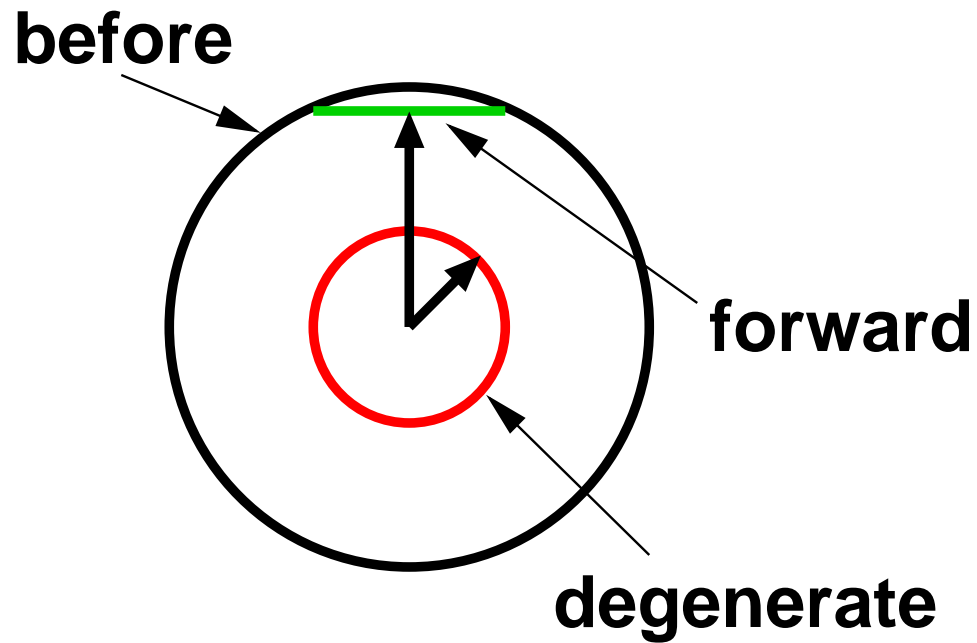
It must choose the forward sphere





$\sqrt{\text{Noise}} = \text{degenerate sphere radius}$

**Thermal noise determines
the radius of the degenerate sphere**

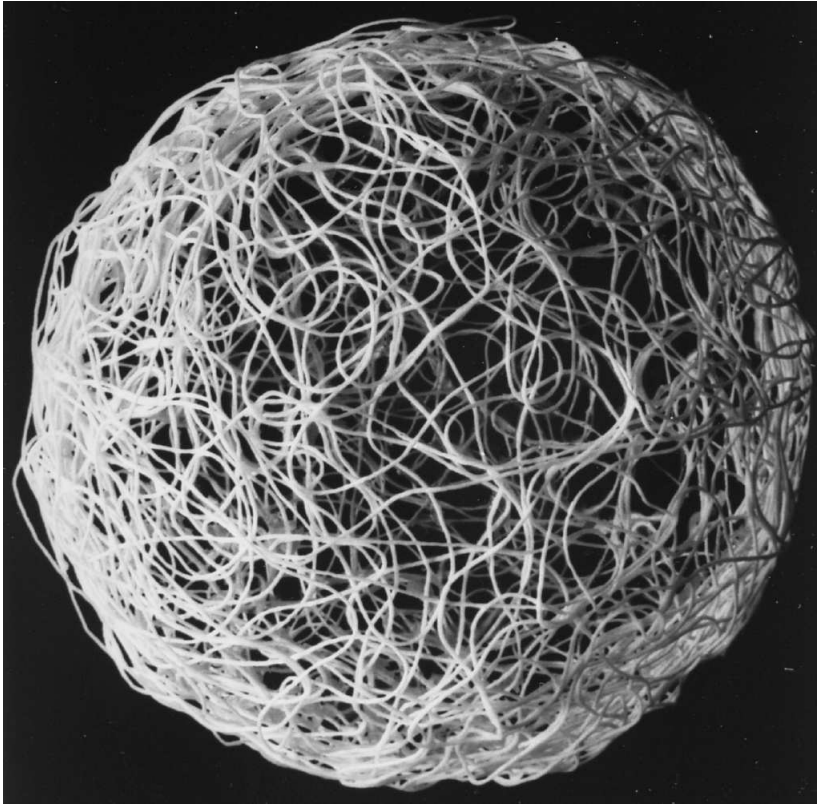


**Criterion for distinct states:
forward does not touch degenerate**

$$\sqrt{\text{Power}} > \sqrt{\text{Noise}}$$

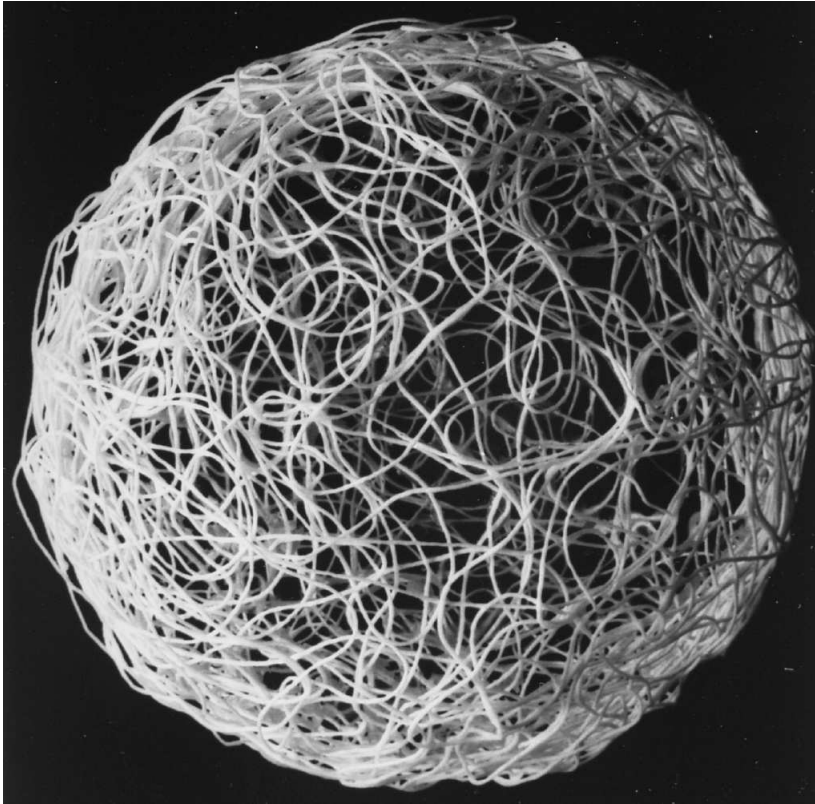
N Dimensional Sphere Separation

Degenerate Sphere

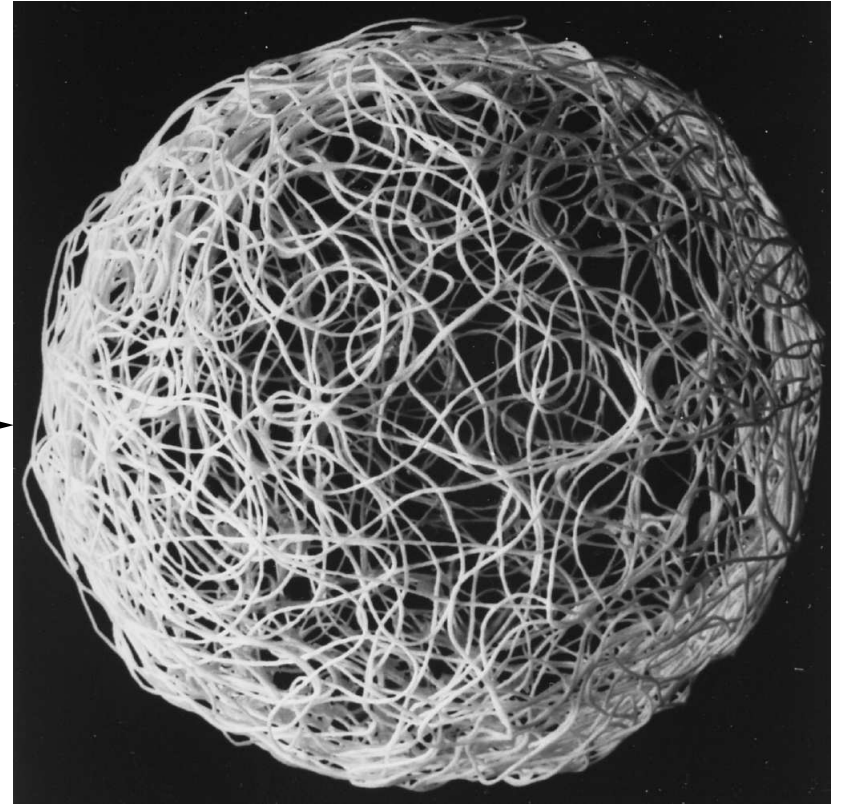


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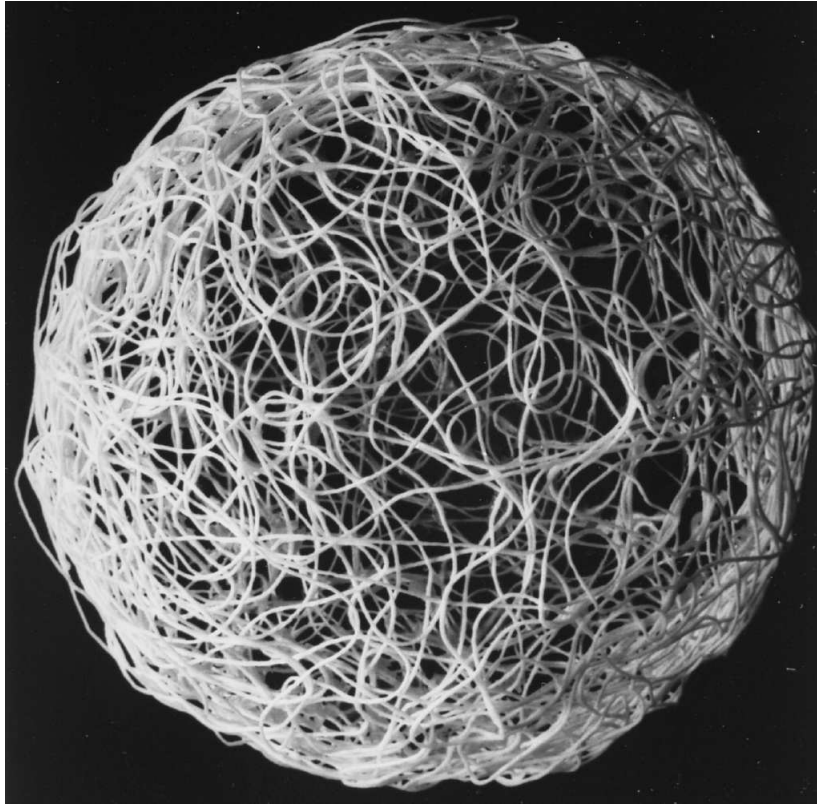


Forward Sphere

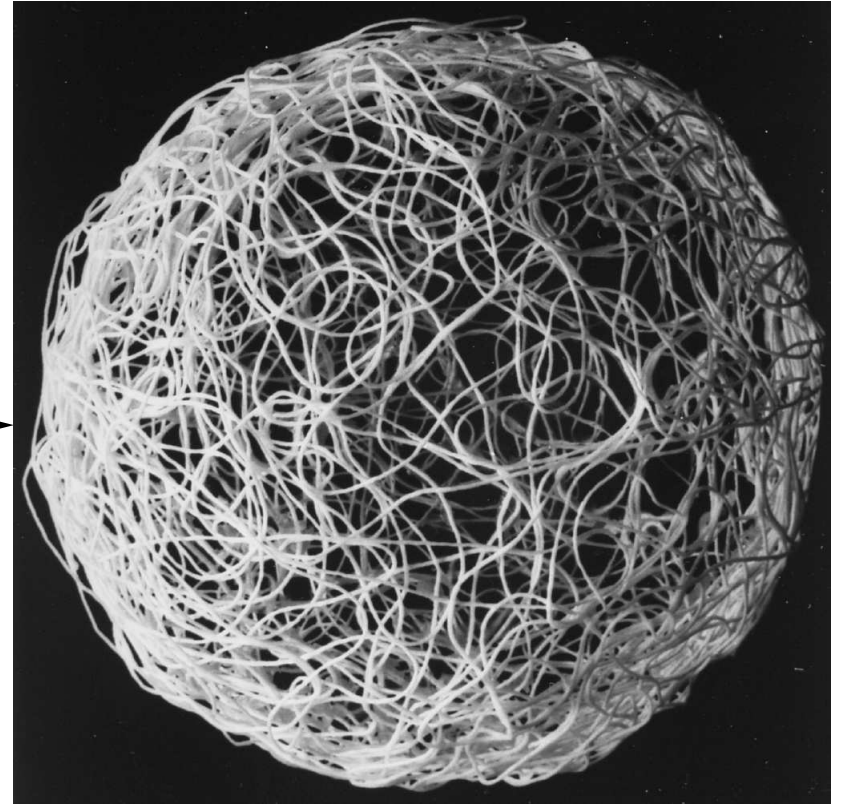


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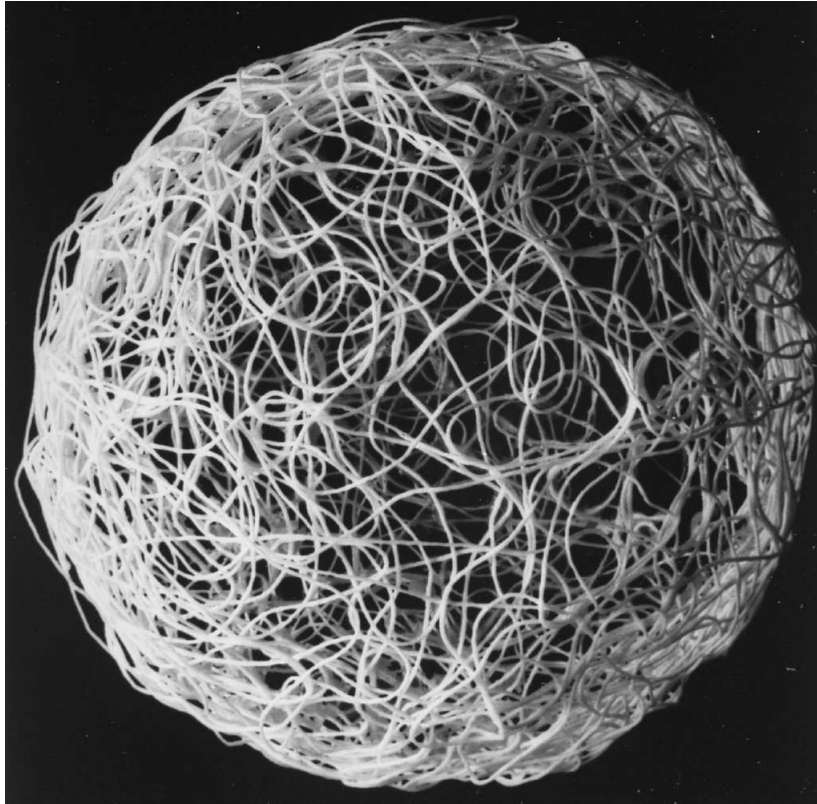
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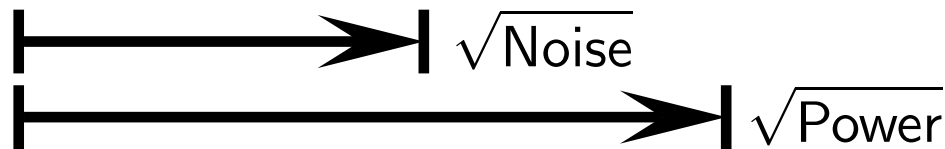
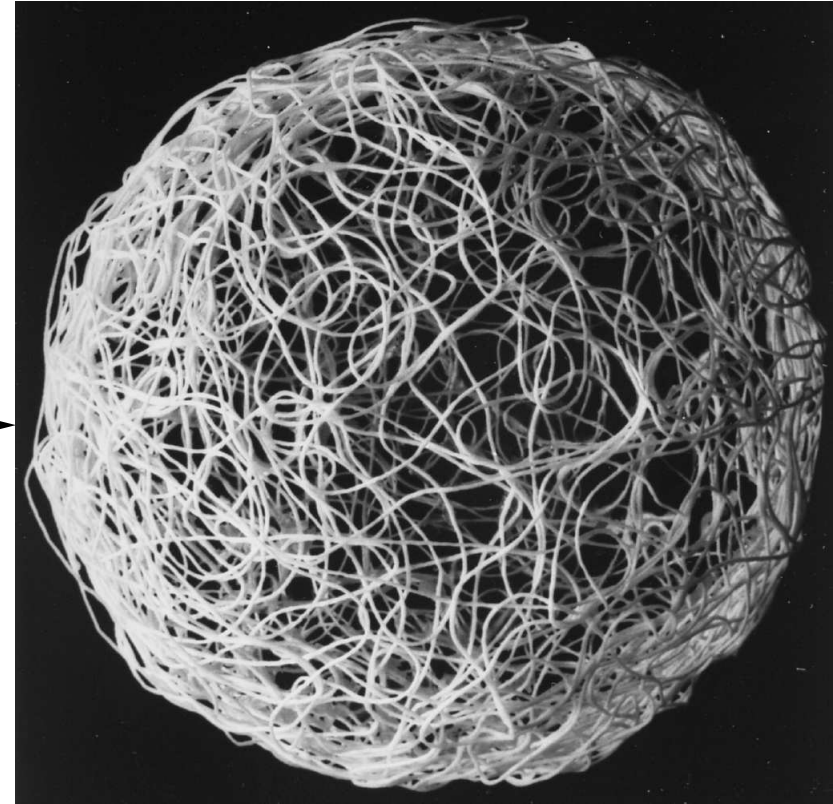
$\sqrt{\text{Noise}}$

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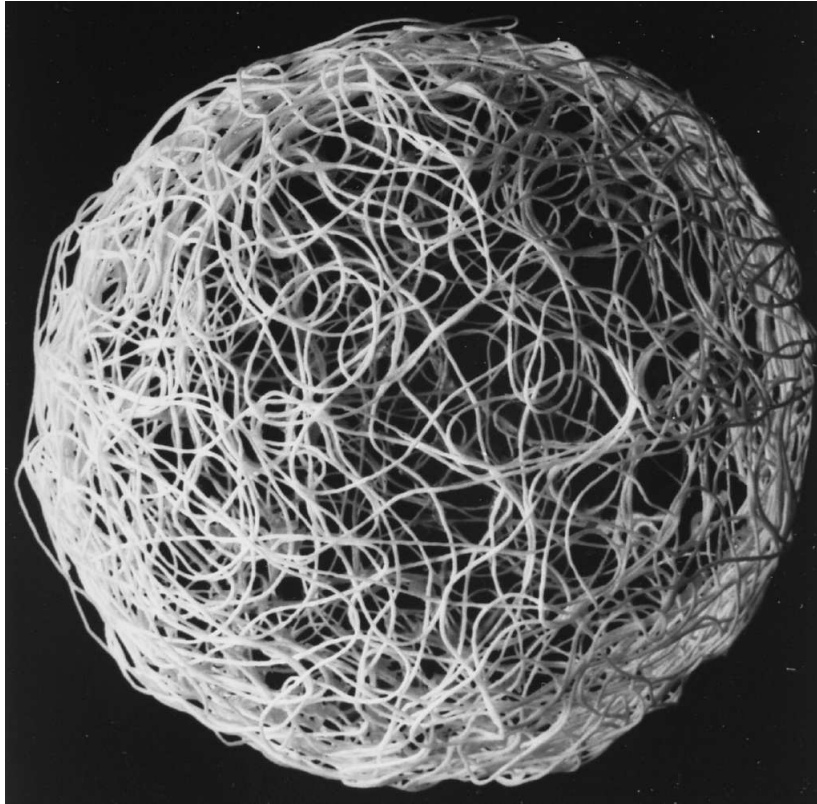


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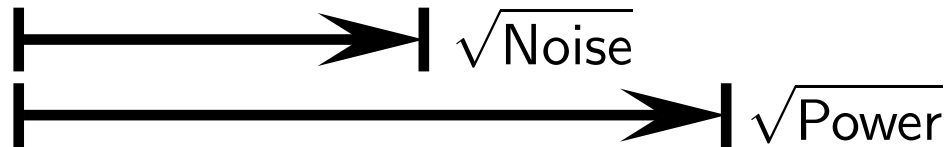
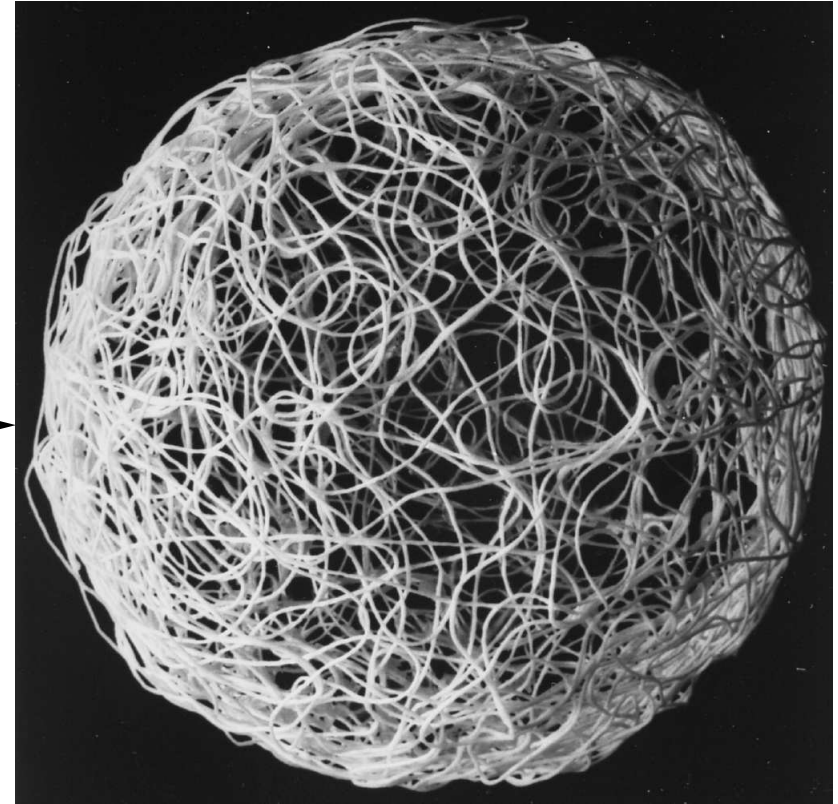


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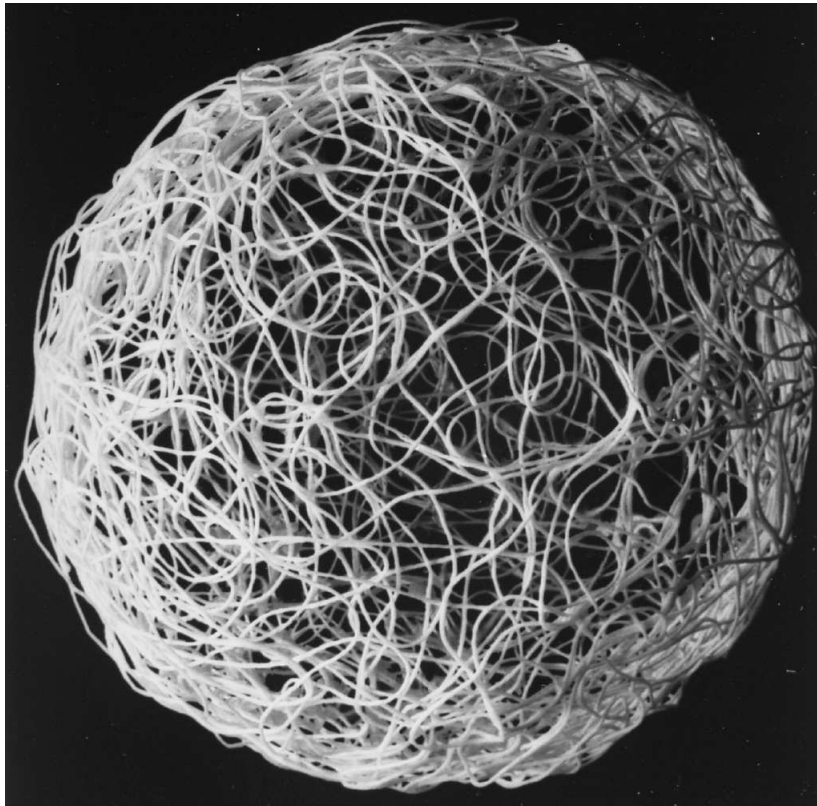
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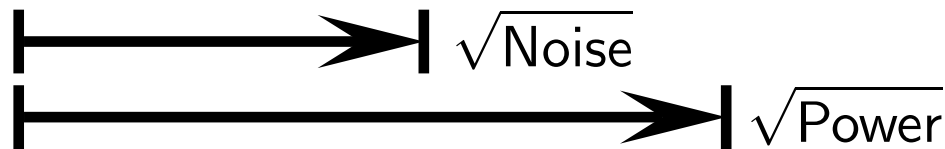
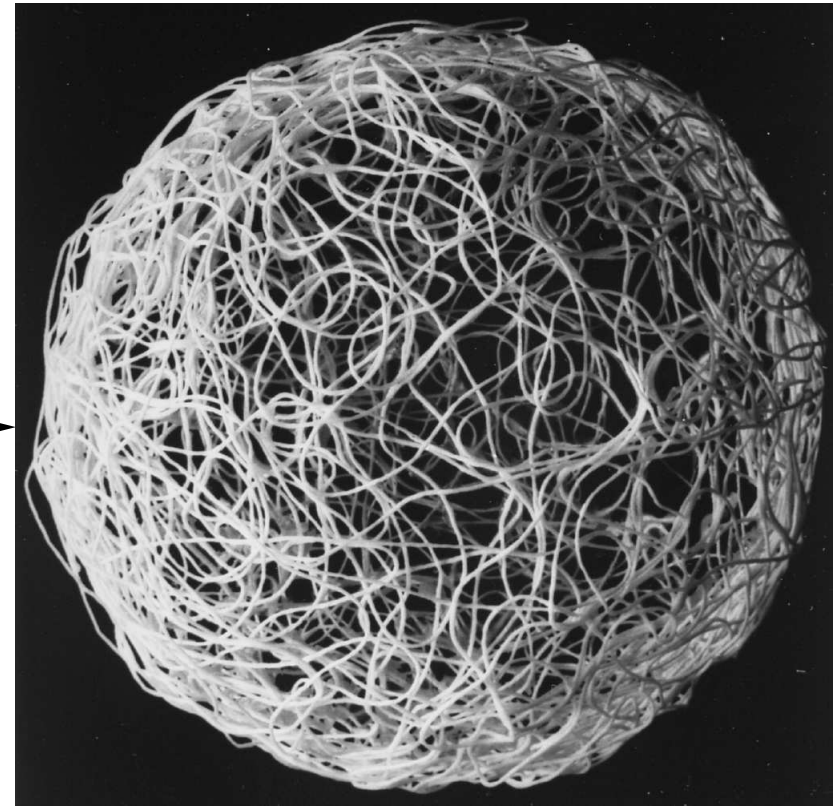
Energy dissipated to escape the Degenerate Sphere must exceed the Noise

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CONSEQUENCES OF THE DEGENERATE SPHERE HYPOTHESIS

The geometry gives:

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T. D. Schneider
Nucl. Acids Res. **2010**
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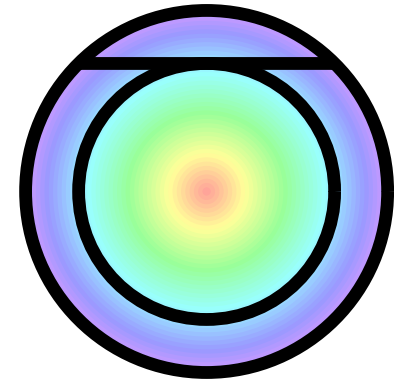
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Why is the Genetic Code Degenerate?

The Genetic Code

Second base in codon

U C A G

U

Phe	Ser	Tyr	Cys	U
Phe	Ser	Tyr	Cys	C
Leu	Ser	och	opa	A
Leu	Ser	amb	Trp	G

C

Leu	Pro	His	Arg	U
Leu	Pro	His	Arg	C
Leu	Pro	Gln	Arg	A
Leu	Pro	Gln	Arg	G

A

Ile	Thr	Asn	Ser	U
Ile	Thr	Asn	Ser	C
Ile	Thr	Lys	Arg	A
Met	Thr	Lys	Arg	G

G

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64 codons

$$\log_2 64 = 6 \text{ bits/amino acid}$$

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A		Ile	Thr	Asn	Ser	U
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Efficiency of The Genetic Code

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Compute Efficiency

$$\begin{aligned} \epsilon_r &= \frac{\log_2 \text{actual choices}}{\log_2 \text{maximum choices}} \\ &= \frac{4.3}{6} = 0.72 \end{aligned}$$

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Phe Ser Tyr Cys U
Phe Ser Tyr Cys C
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Leu Ser **amb** Trp G

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Leu Pro Gln Arg G

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The Genetic Code fits the theory!

Amino Acid Frequencies

A	91298299
C	15183770
D	59081152
E	67663968
F	42689961
G	72802737
H	23851938
I	61214309
K	57561410
L	104783181
M	24024396
N	46921121
O	5
P	53406141
Q	43463766
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T	60736608
U	301
V	70111092
W	13441284
Y	32887204

Refine the Calculation

Obtain actual amino acid frequencies from the 50% sequence identity non-redundant Protein Information Resource (PIR) UniRef50 database, June 2010.

$$n = 1,083,655,243 = 1.1 \times 10^9 \text{ amino acids}$$

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Compute the uncertainty:

$$\begin{aligned} H_{aa} &= - \sum_{aa = A}^Y P_{aa} \log_2 P_{aa} \quad \text{bits per amino acid} \\ &= 4.1706 \quad \text{bits per amino acid} \end{aligned}$$

That's what is actually accomplished by translation.

Translational Efficiency

Compute the efficiency:

$$\epsilon_r = \frac{4.1706}{6}$$

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Compute the efficiency:

$$\begin{aligned}\epsilon_r &= \frac{4.1706}{6} \\ &= 0.6951 \text{ Measured efficiency} \\ \epsilon_t &= 0.6931 \text{ Theoretical maximum} = \ln(2) \\ &\quad 0.0020 \text{ difference}\end{aligned}$$

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0.2% excess is significant!

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Efficiency of the Genetic Code

Theory Violation! What's missing?

Error rate of transcription/translation was not accounted for.

See if we can compute it.

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Average probability of misincorporation, P_{error} determines the information lost:

$$H_{\text{error}} = [-P_{\text{error}} \log_2 P_{\text{error}}] + [-(1 - P_{\text{error}}) \log_2 (1 - P_{\text{error}})]$$

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$$\epsilon_r = \frac{H_{\text{before}} - H_{\text{after}}}{6} = \frac{4.1706 - H_{\text{error}}}{6} = \ln 2$$

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Solving gives the **theoretically predicted error rate of translation**:

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Error rate of transcription/translation was not accounted for.
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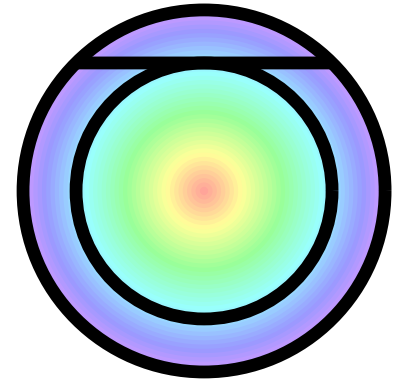
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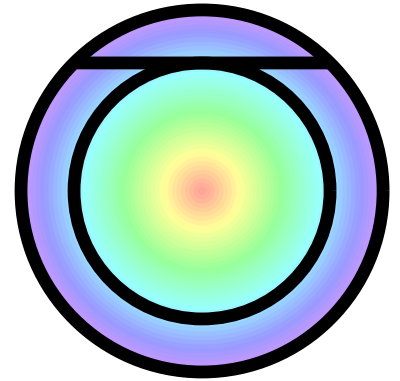
The theory matches the data to 4 decimal places!

- Establishes a novel mathematical field of biology



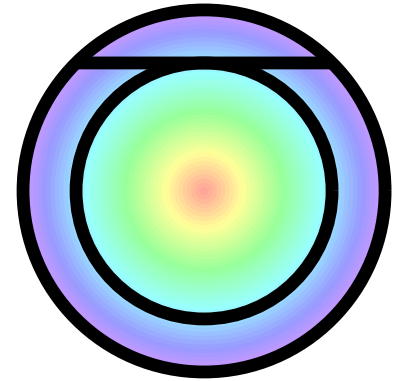
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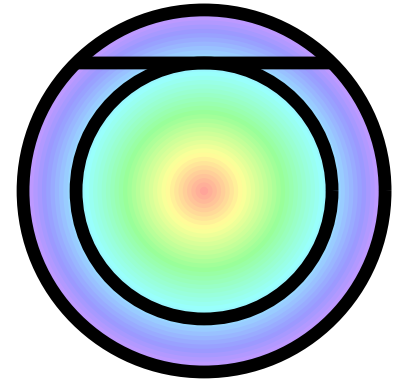
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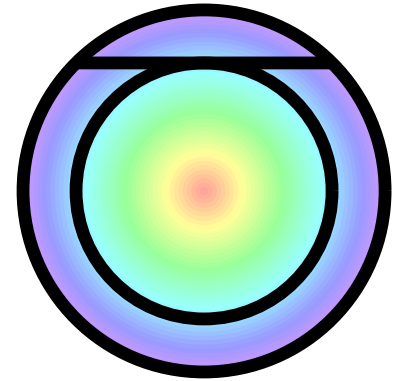
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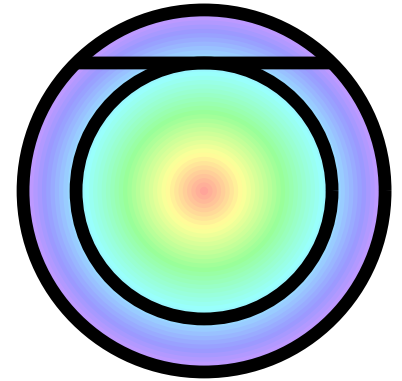
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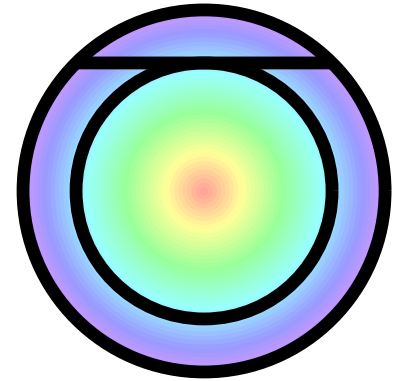
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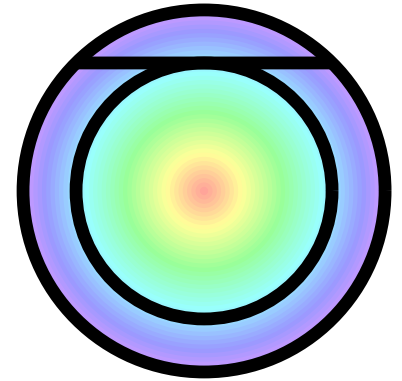
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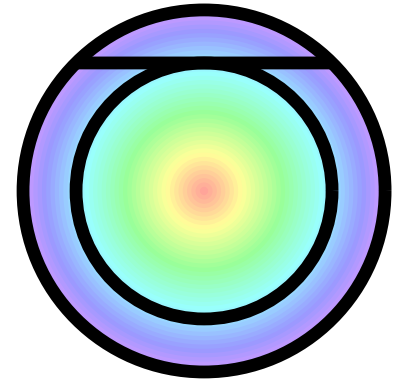
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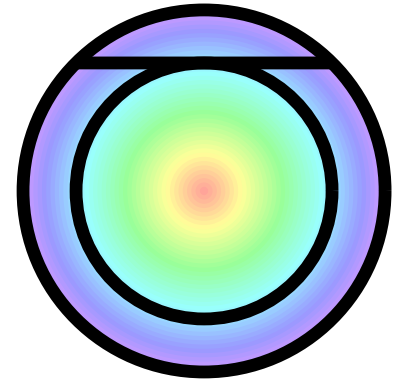
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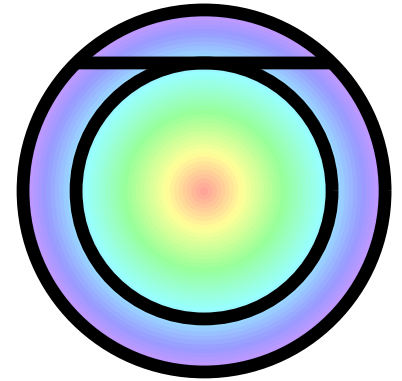


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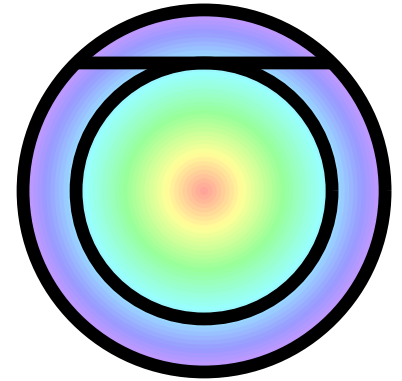


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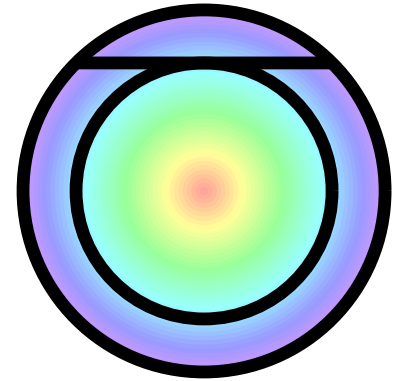
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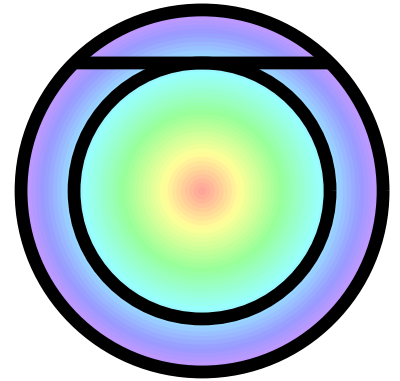
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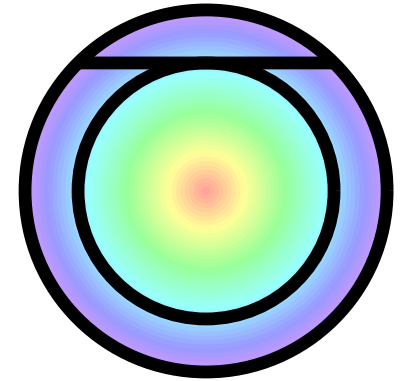
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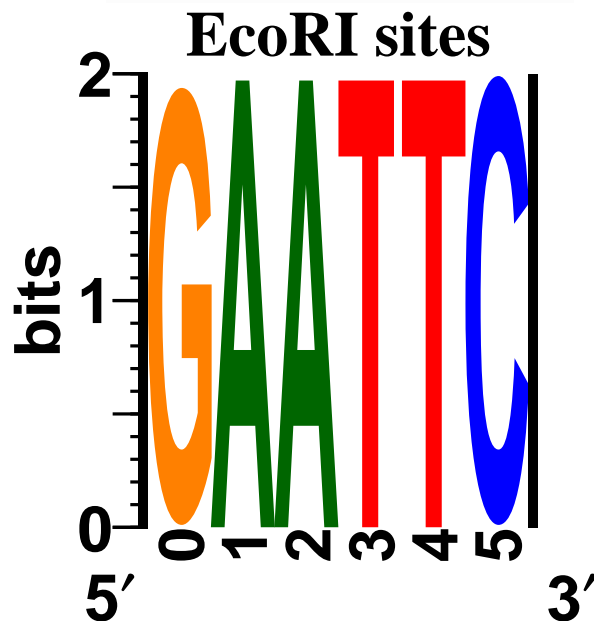
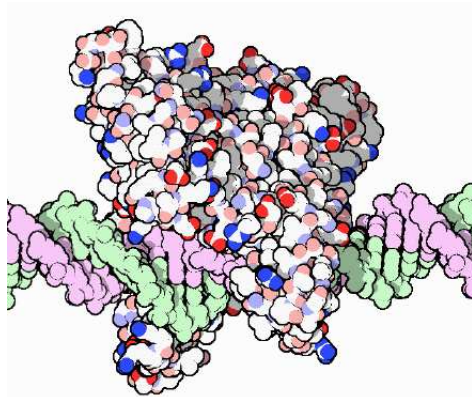
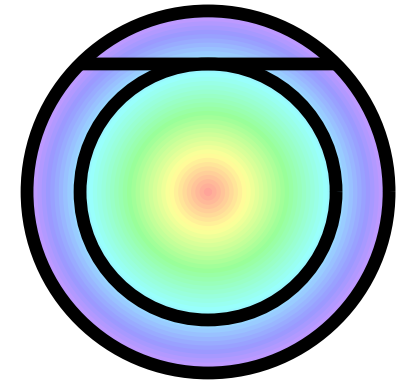
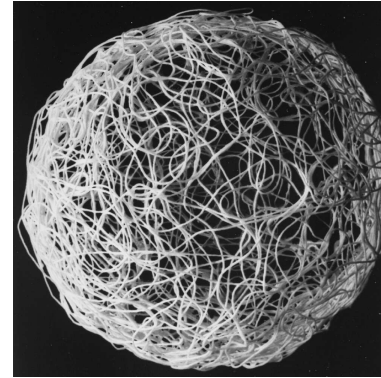
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National Institutes of Health, National Cancer Institute



Web site:
TinyURL.com/tomschneider



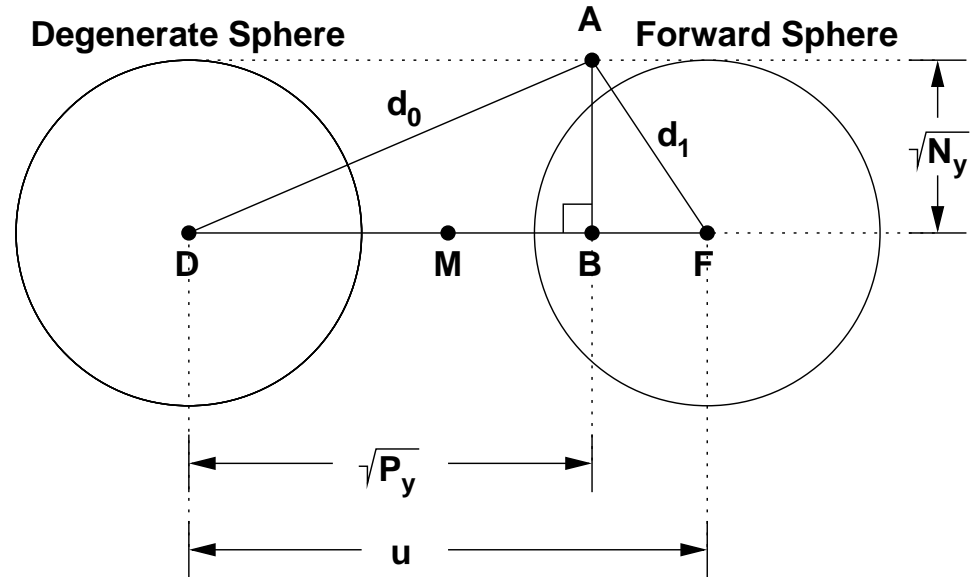
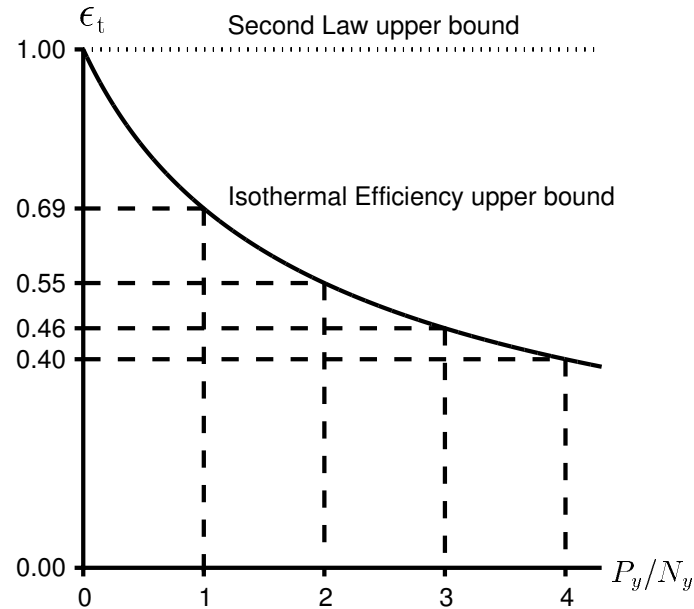
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Version

version = 1.37 of hidimtalk.tex 2010 Aug 05

Proof that $P_y > N_y$, $\epsilon < \ln(2)$



buffer zone:

$$u > 2\sqrt{N_y} \quad (0)$$

distance² from A to D:

$$d_0^2 = \sqrt{P_y}^2 + \sqrt{N_y}^2 = P_y + N_y \quad (1)$$

distance² from A to F:

$$d_1^2 = (u - \sqrt{P_y})^2 + \sqrt{N_y}^2 \quad (2)$$

decoding to forward sphere:

$$d_1 < d_0 \quad (3)$$

(1) and (2) into square of (3):

$$\sqrt{P_y} > u/2 \quad (4)$$

from (0) and (4):

$$\sqrt{P_y} > \sqrt{N_y} \quad \text{so} \quad P_y > N_y \quad (5)$$

$$\epsilon = \frac{\ln\left(\frac{P_y}{N_y} + 1\right)}{\frac{P_y}{N_y}} \quad \text{so} \quad \epsilon < \ln(2) \approx 0.6931$$

An Intuitive Approach

Information to chose one symbol from M symbols:

$$\log_2 M \quad (6)$$

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If the probabilities P_i of different symbols, i , are not equal, then the **surprisal** is:

$$u_i \equiv -\log_2 P_i.\tag{7}$$

how surprised one is to see a symbol

EXAMPLE

A phone rings once every 1024 seconds.



$$P_{\text{ring}} = 1/1024 \quad (8)$$

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$$= - \sum_{i=1}^M P_i \log_2 P_i \quad \text{bits per symbol} \quad (16)$$

Information is a decrease in uncertainty

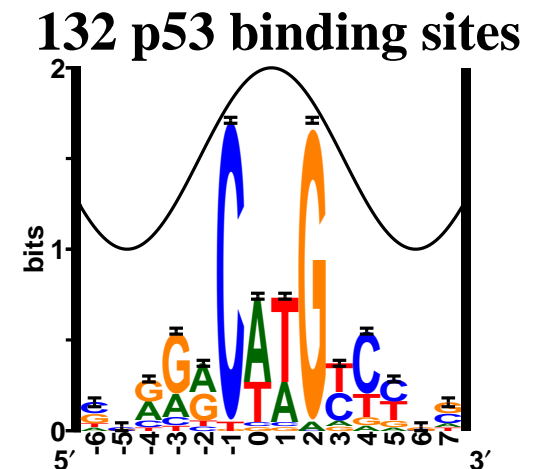
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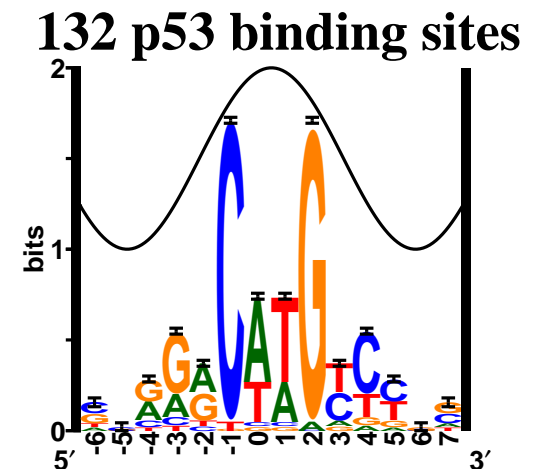
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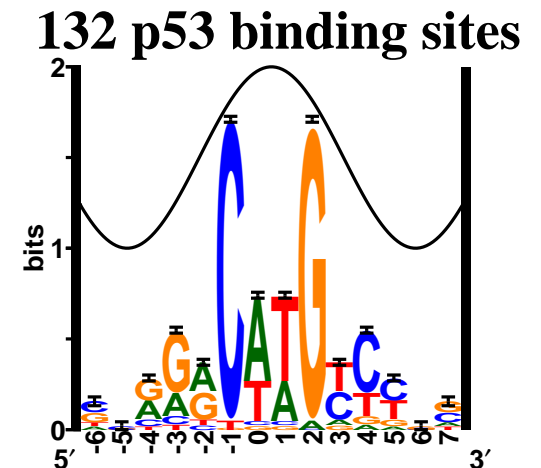
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Note: with only one base, $H_{\text{after}} = 0$
so $R = 2 \text{ bits/base}$.

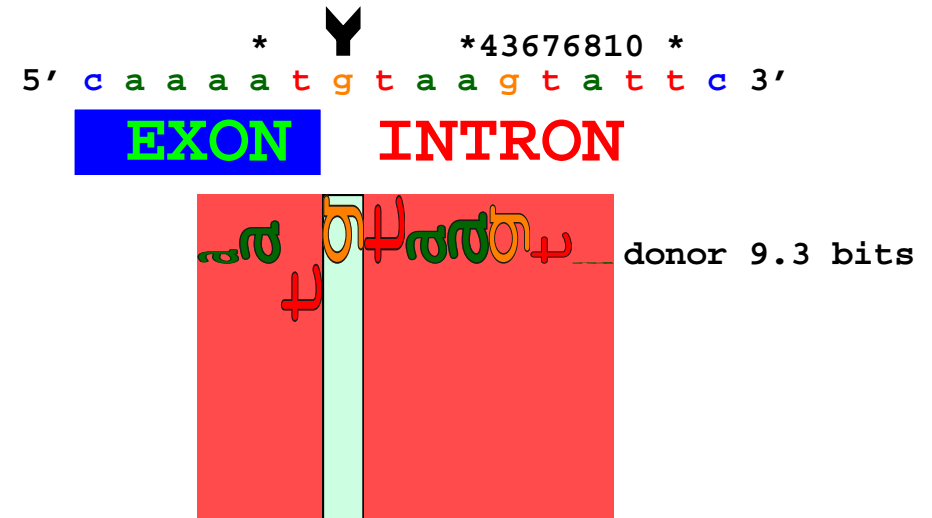


Predicting splicing mutations using information theory

- Xeroderma Pigmentosum-Variant:
defective postreplication repair
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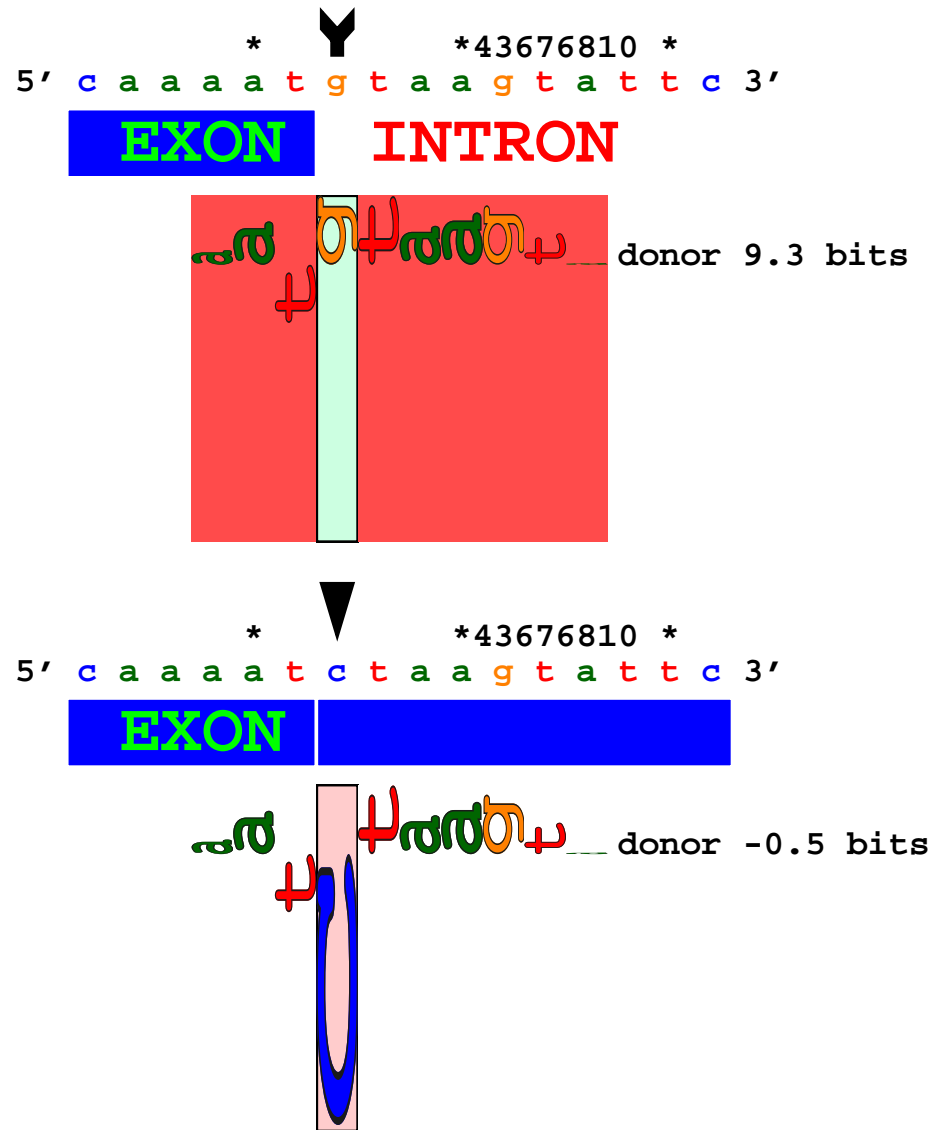
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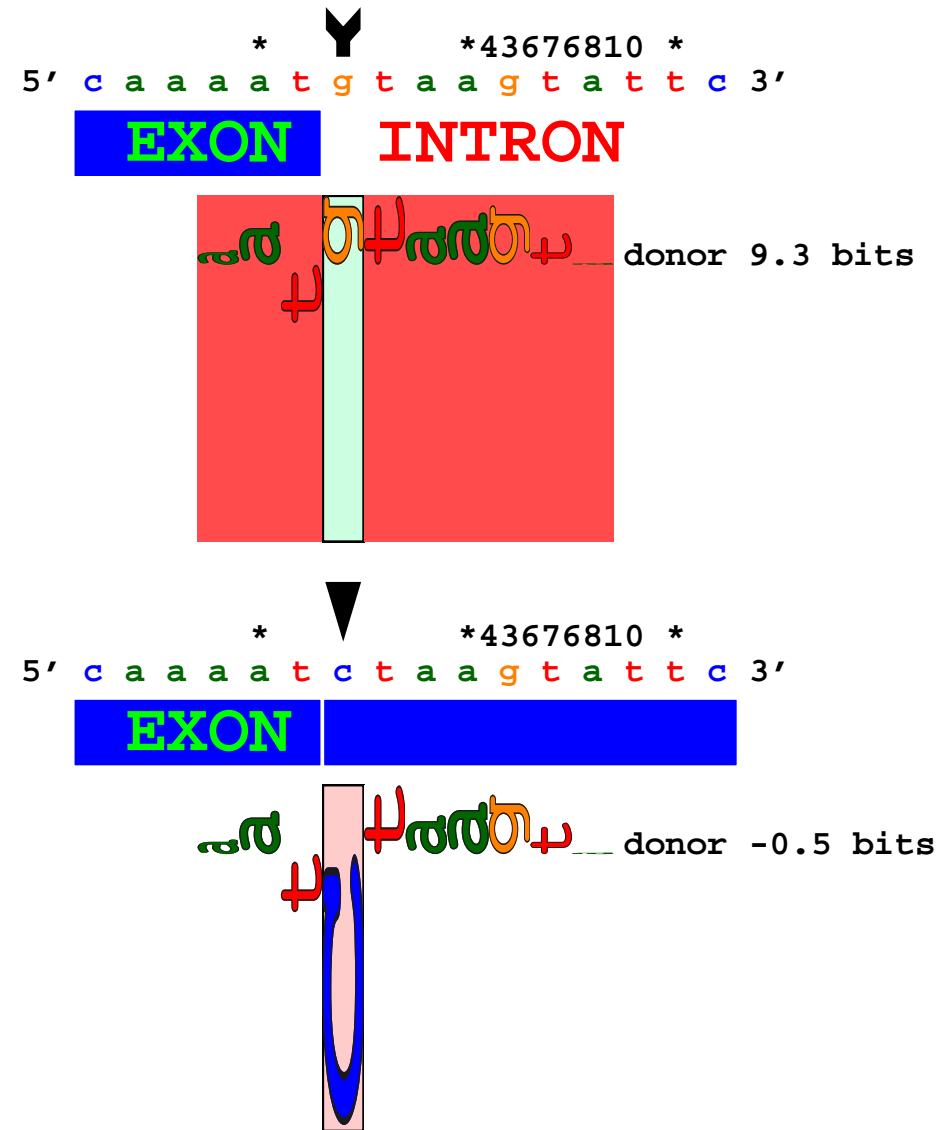
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Inui, . . . , **Schneider** and Kraemer,
J Invest Dermatol. 128:2055-68 (2008)

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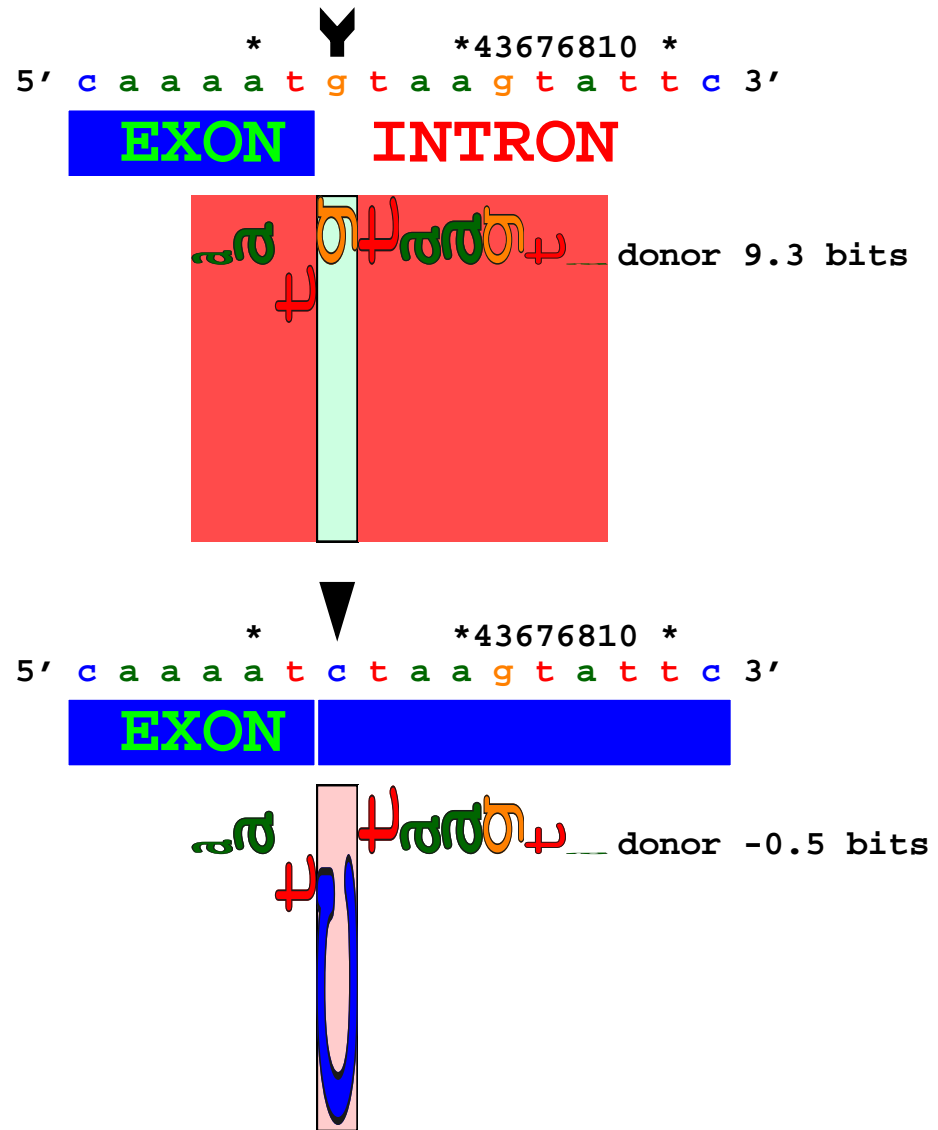
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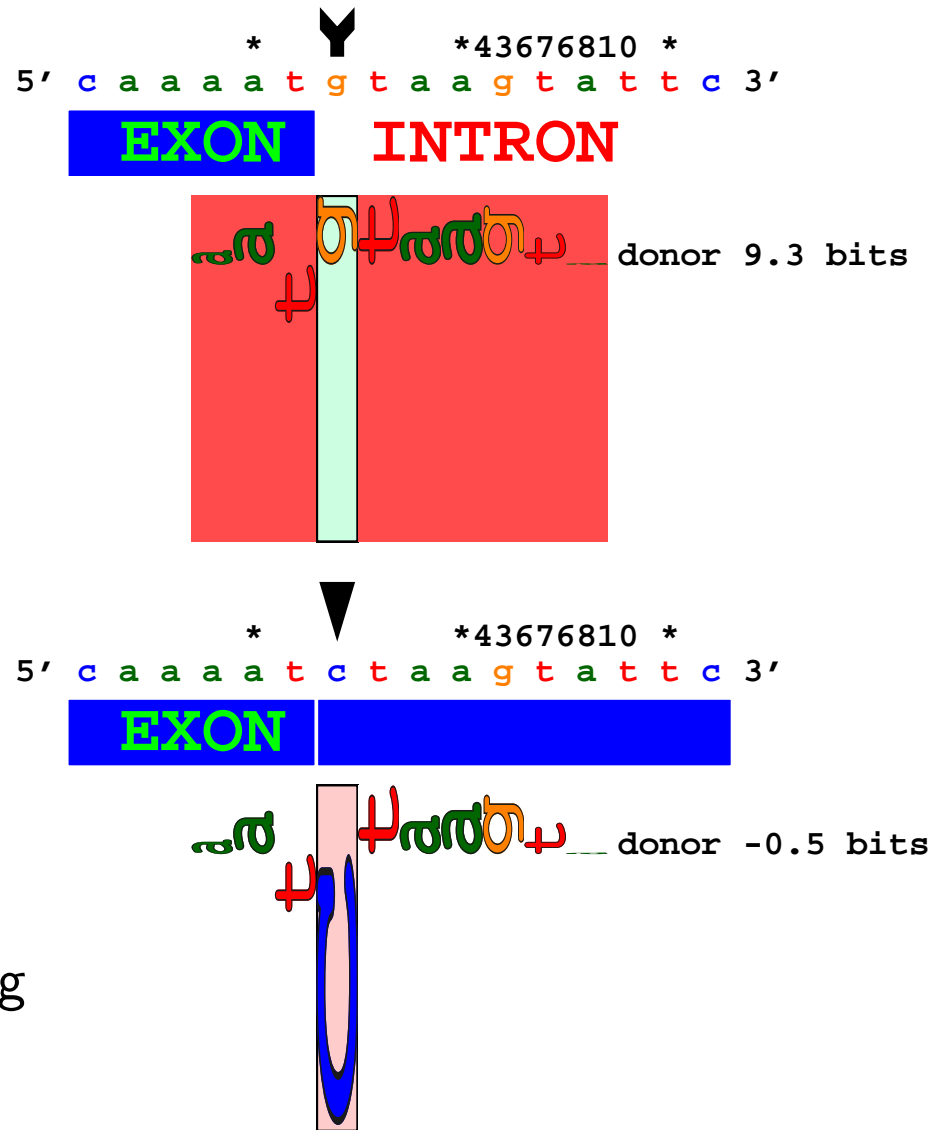
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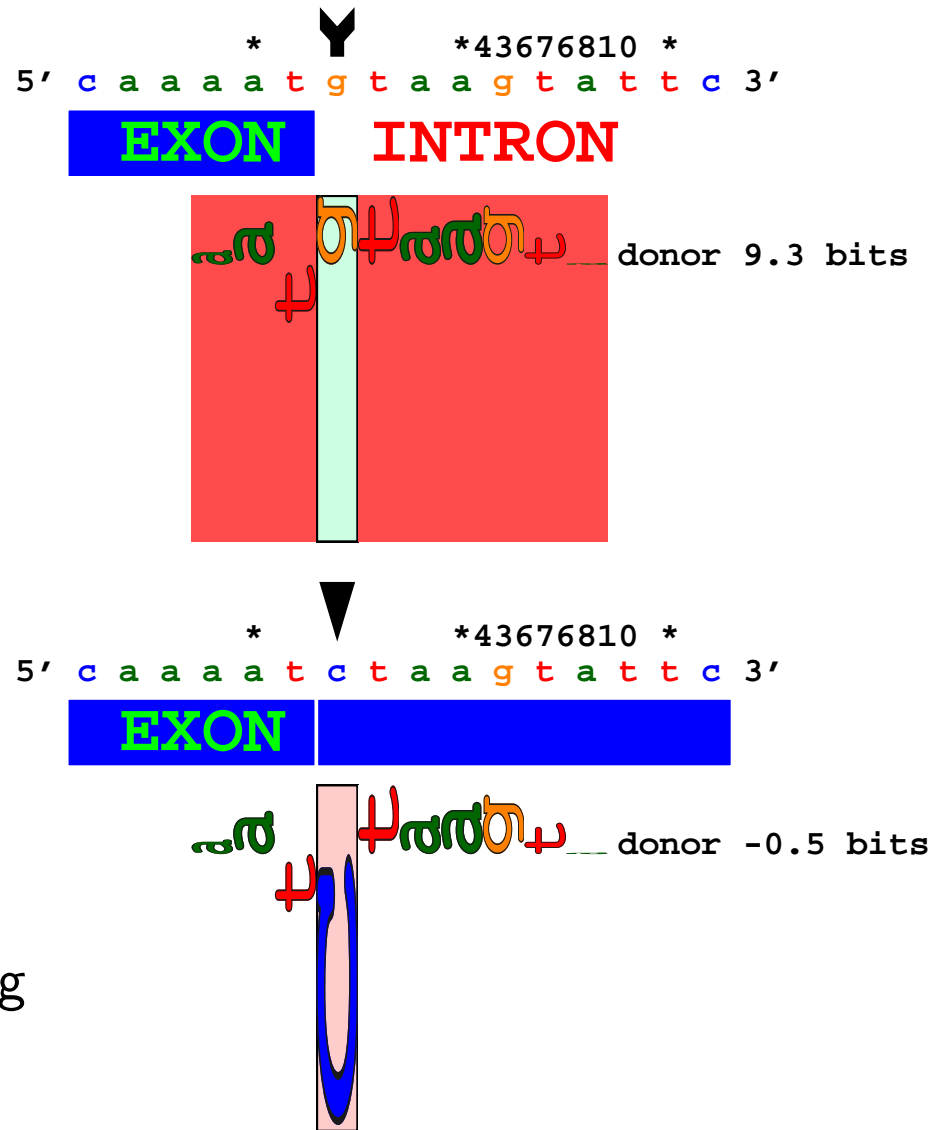
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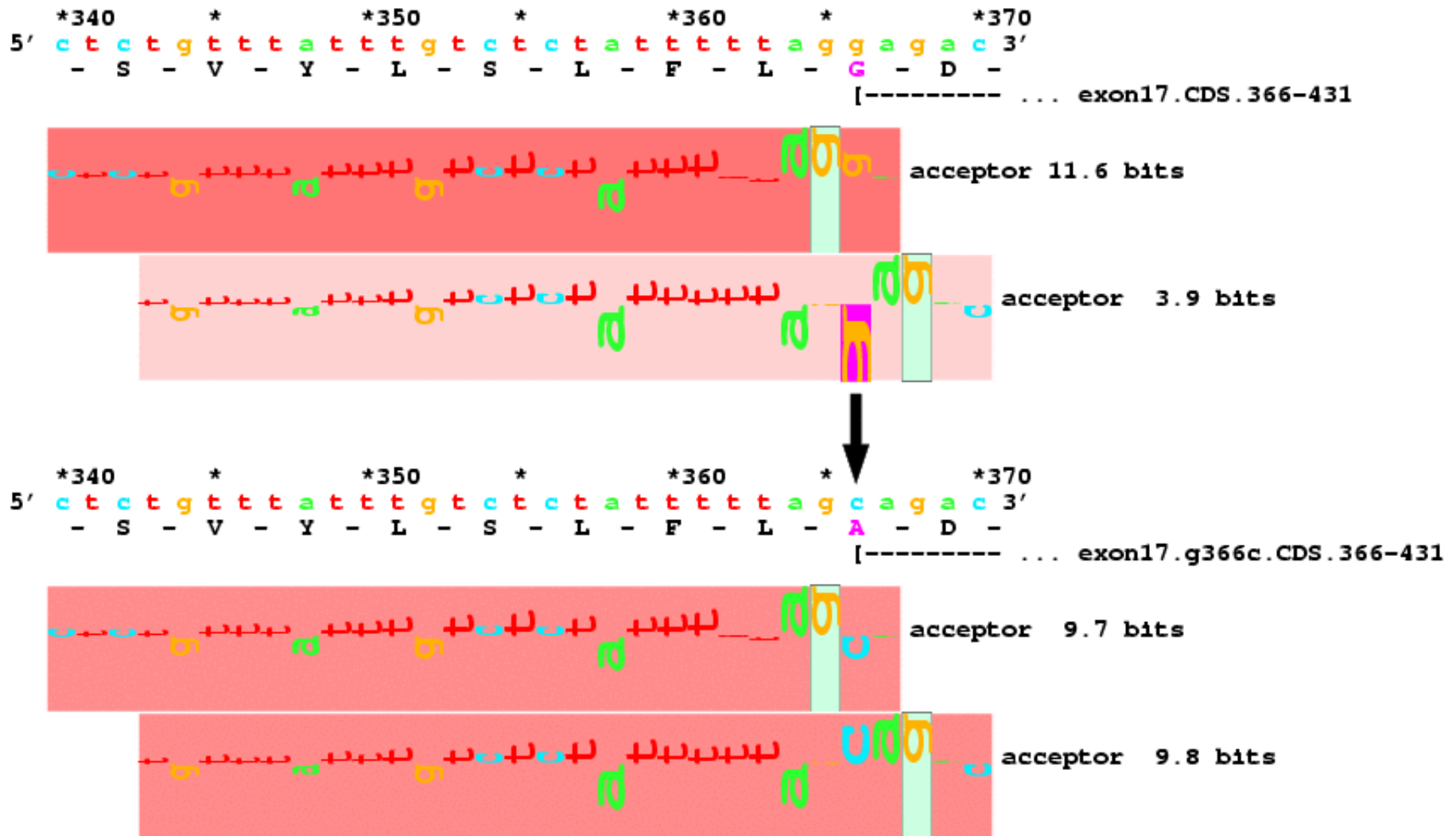
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Medical Applications of Sequence Walkers



Mutation G863A: Stargardt disease = age-related macular degeneration

Discovering p53/p63/p73 controlled genes

- p53 - transcriptional regulator controlling cell cycle

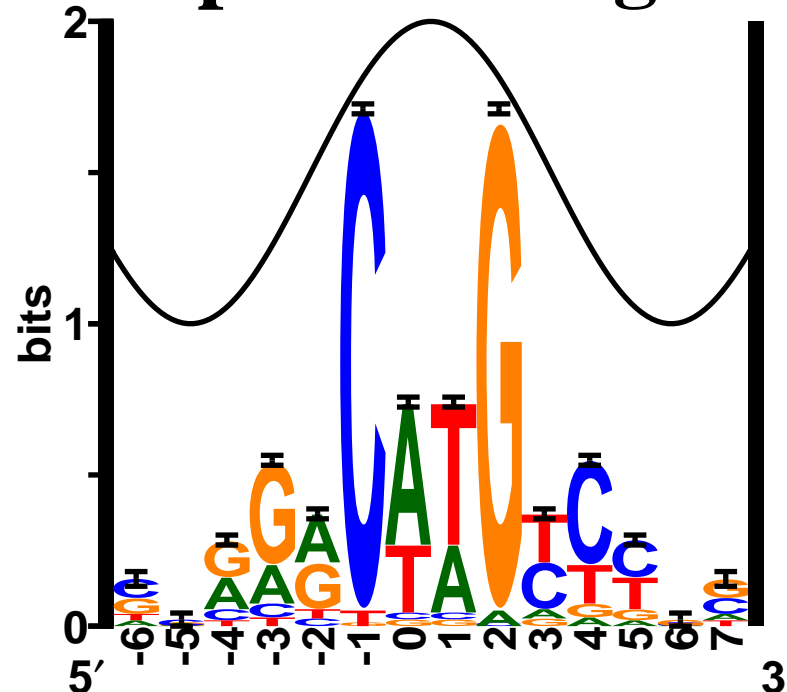
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132 p53 binding sites



Lyakhov, Annangarachari and **Schneider**
Nucleic Acids Res. 36:3828-33 (2008)

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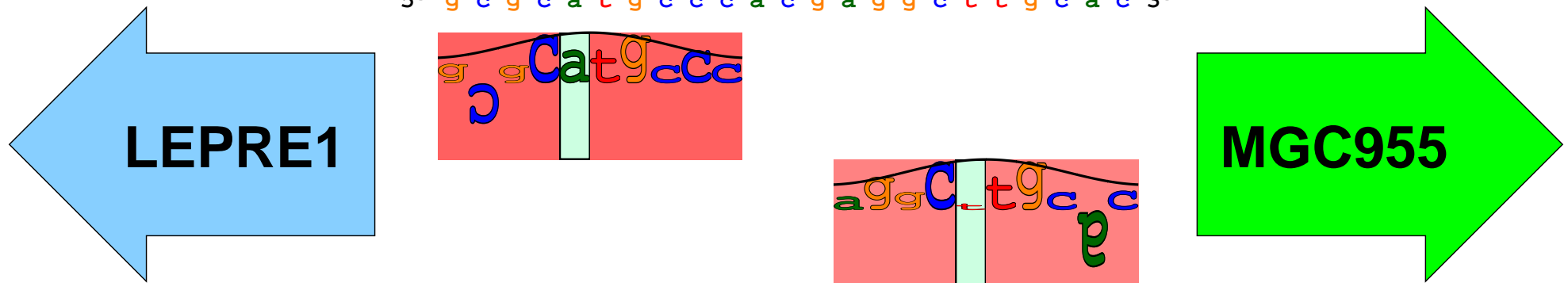
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Discovery of a 7th Bacteriophage λ Operator

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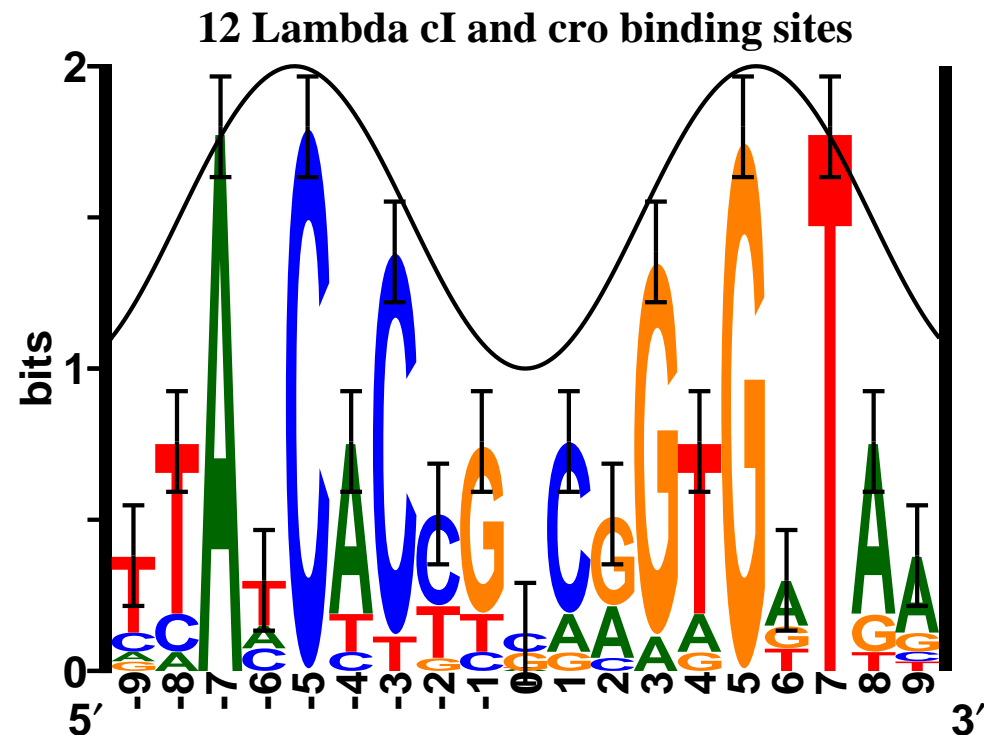
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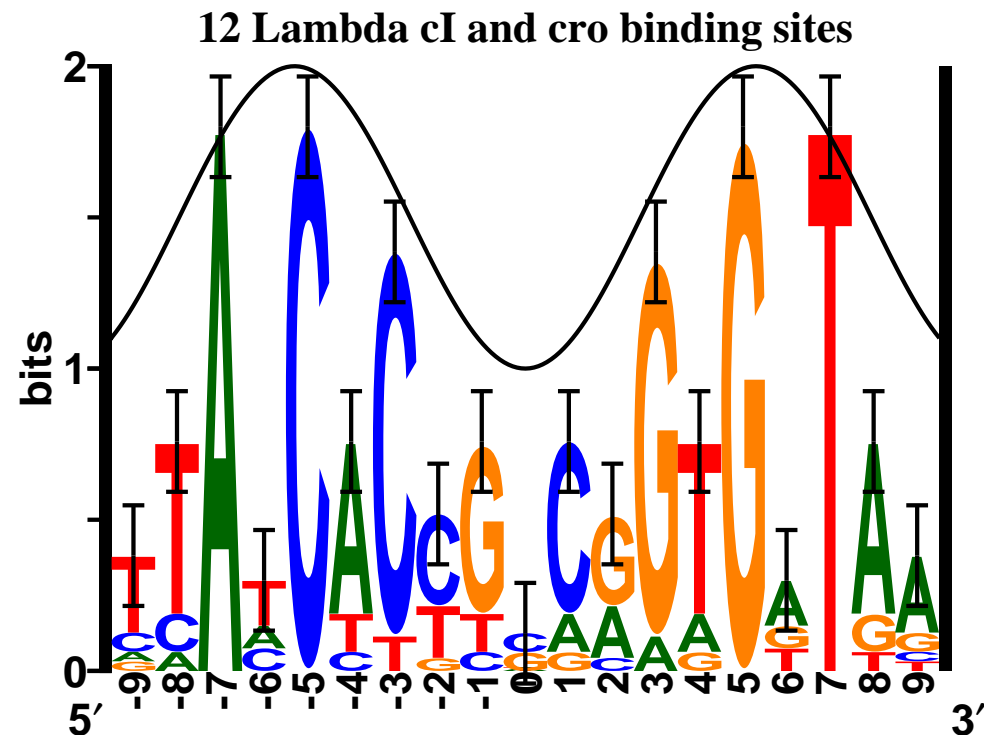


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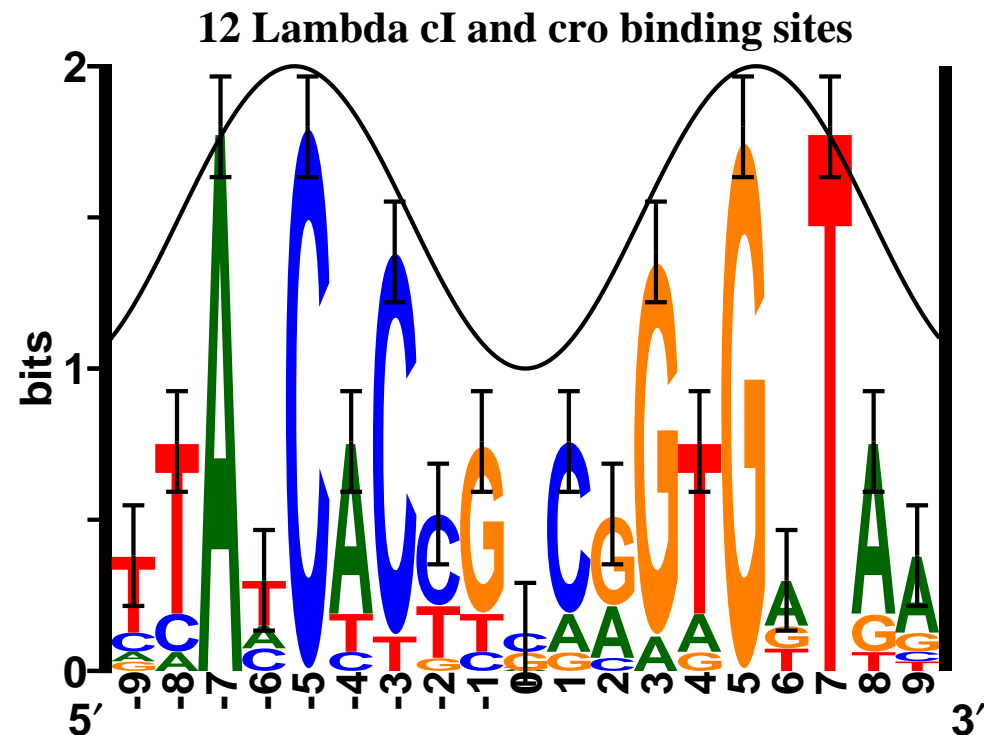
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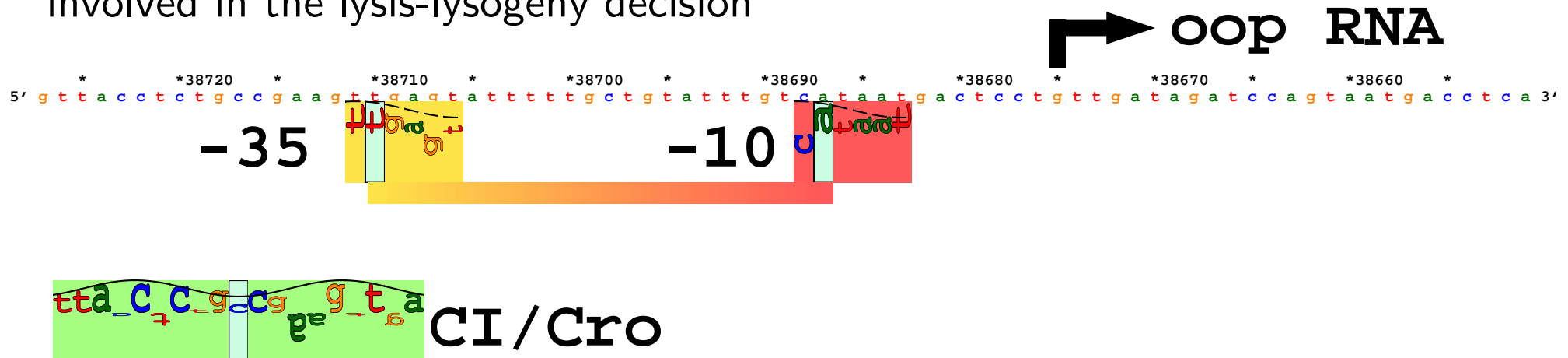
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- Search λ using information theory model
- A 7th Operator found!

Bacteriophage λ Oop promoter: controlled by CI/Cro?

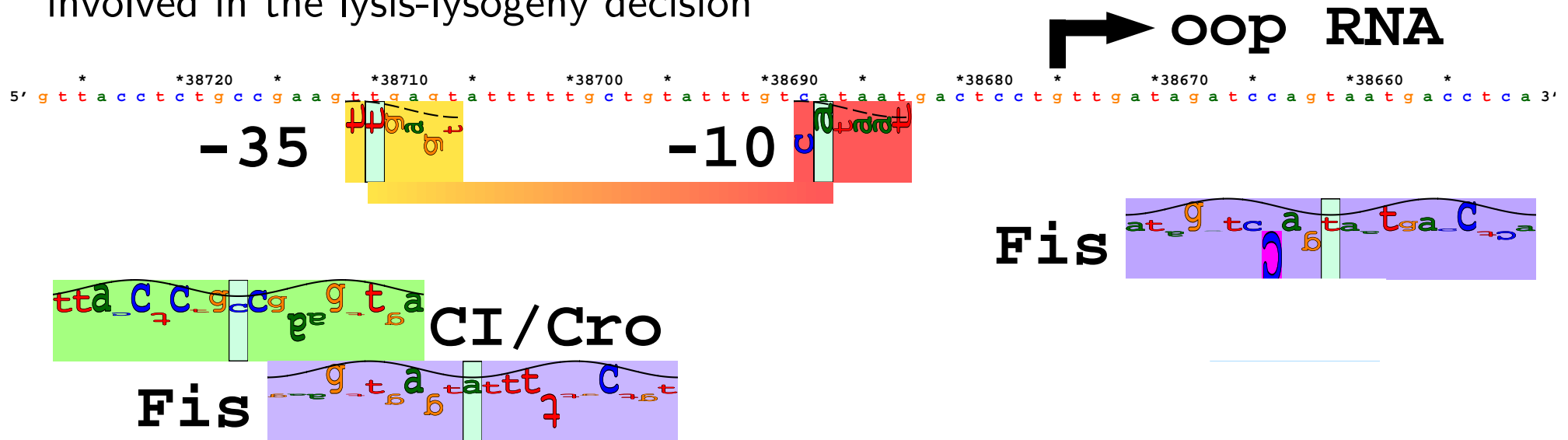
oop RNA is antisense to the 3' end of *cII* mRNA
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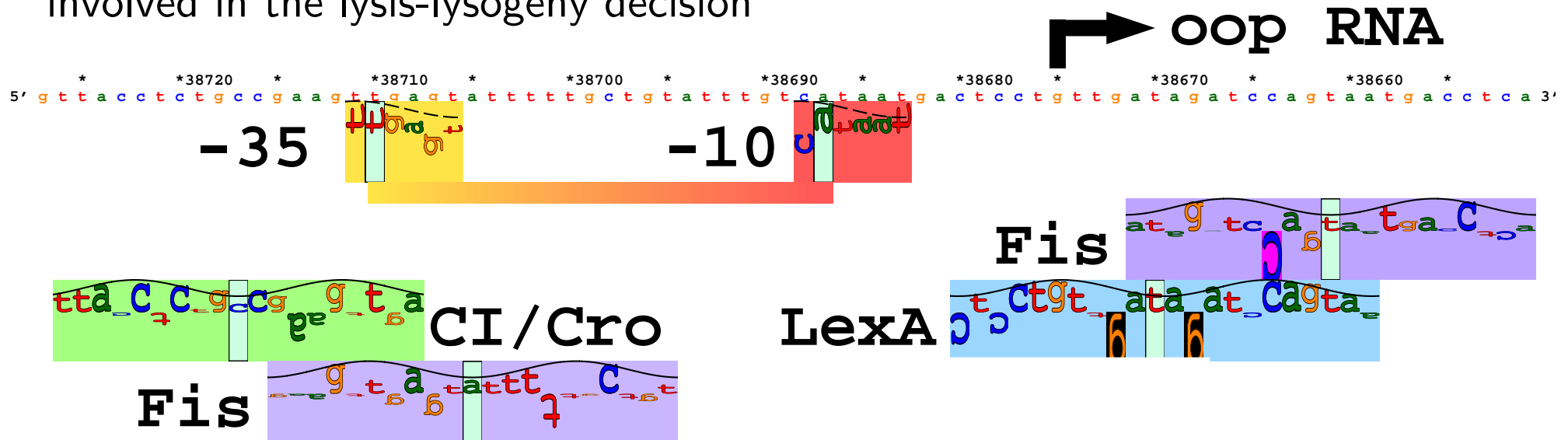
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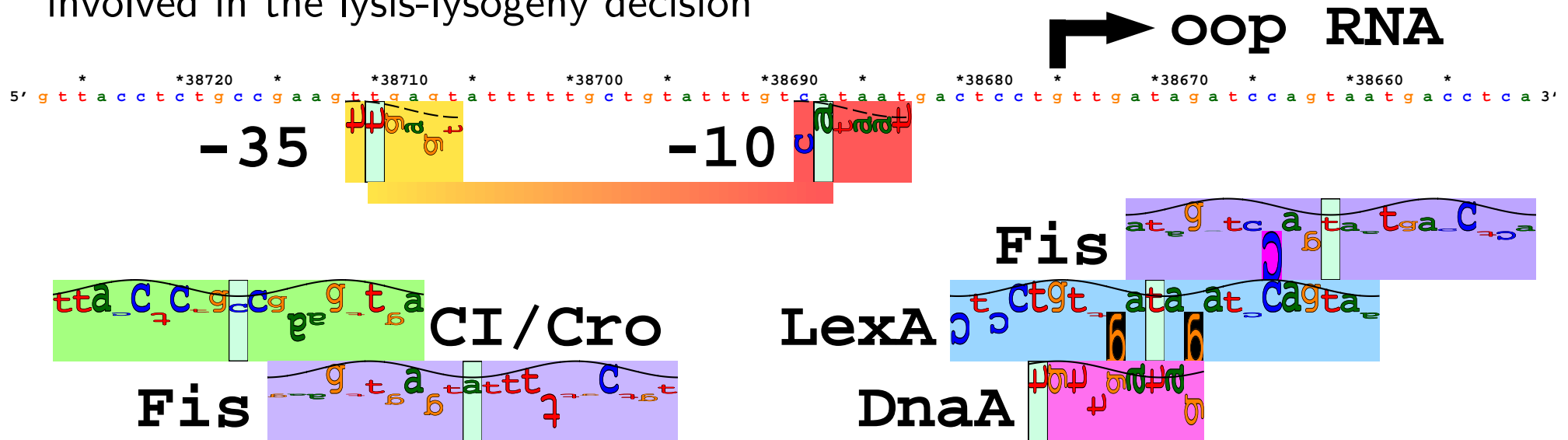
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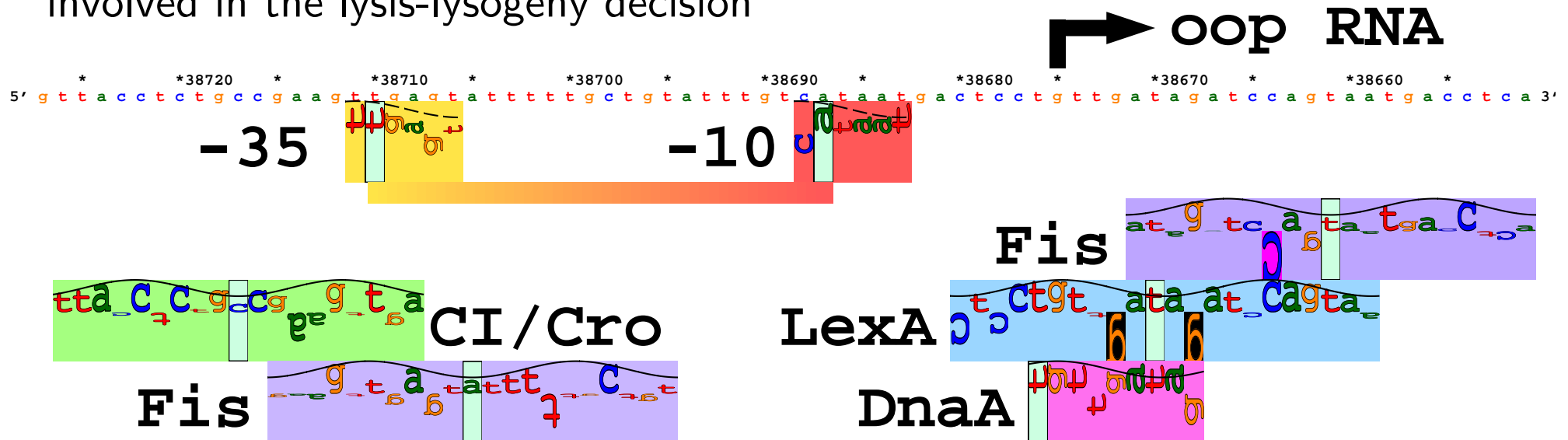


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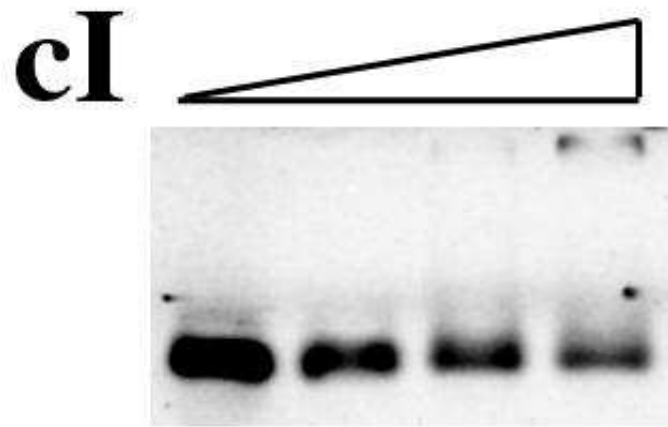
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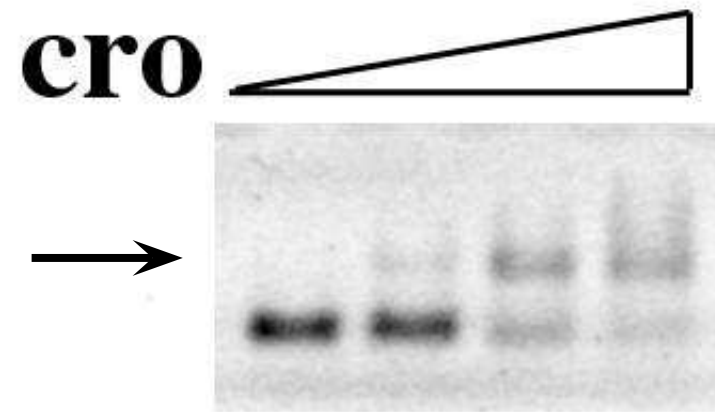
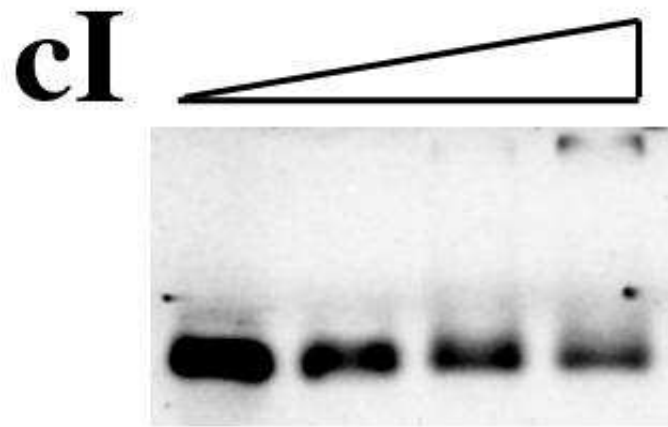
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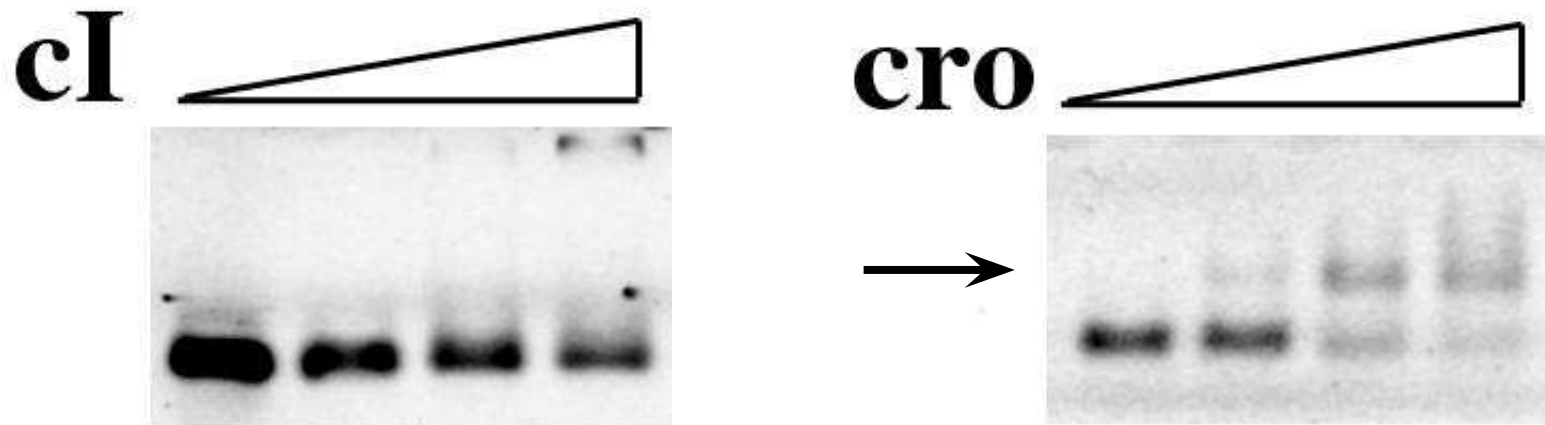


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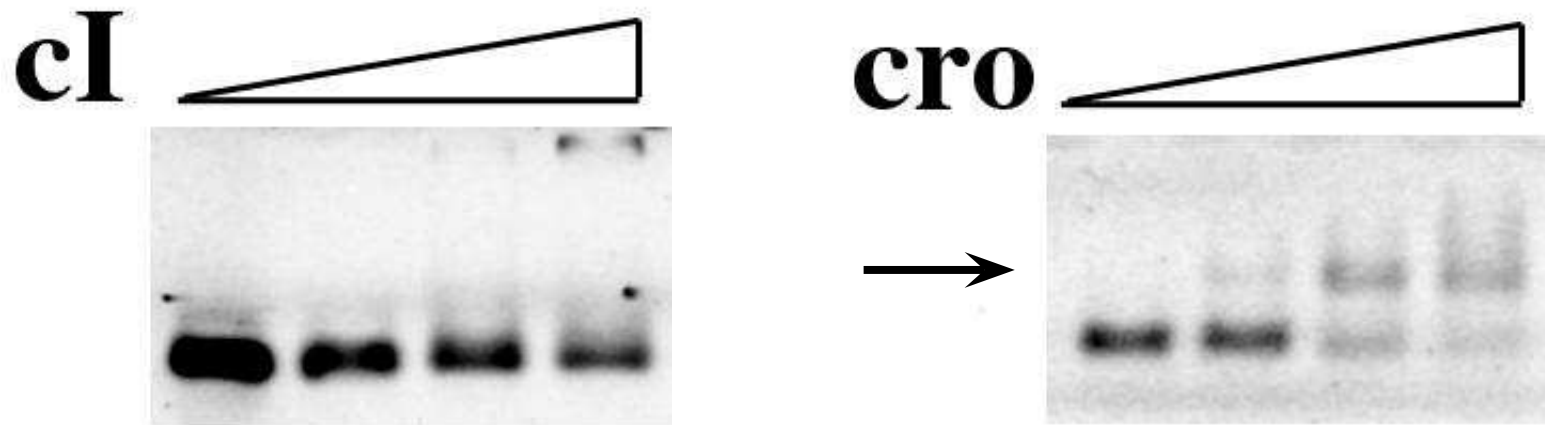
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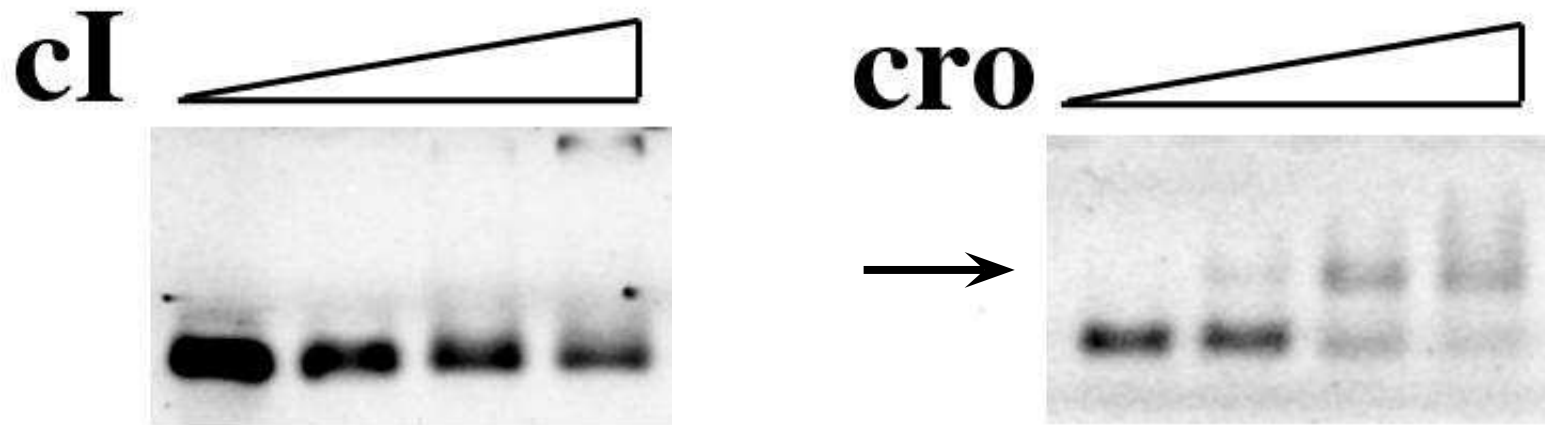




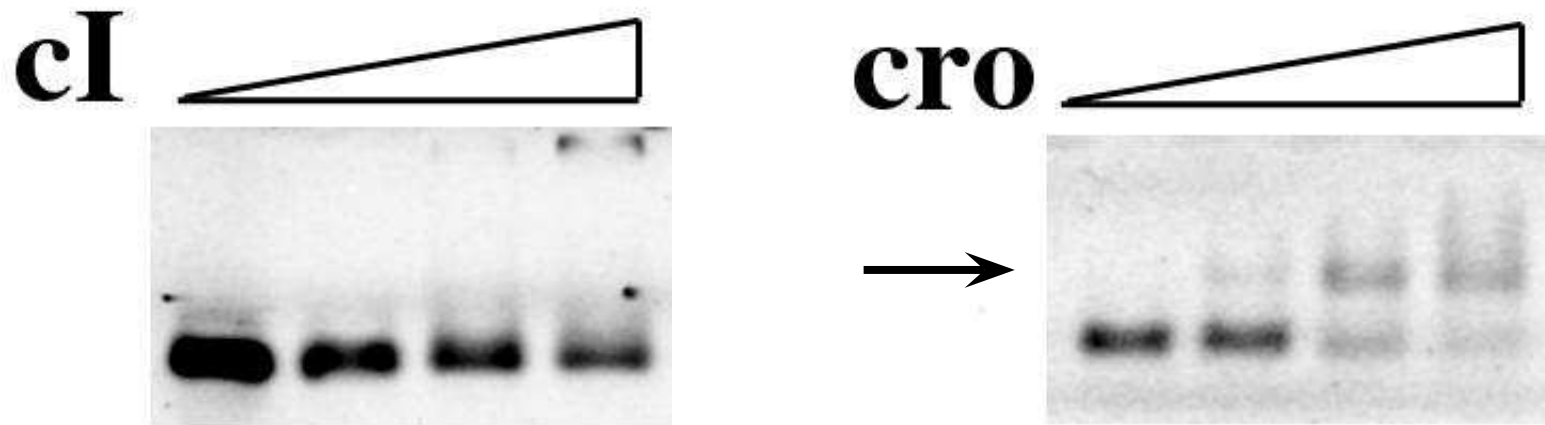
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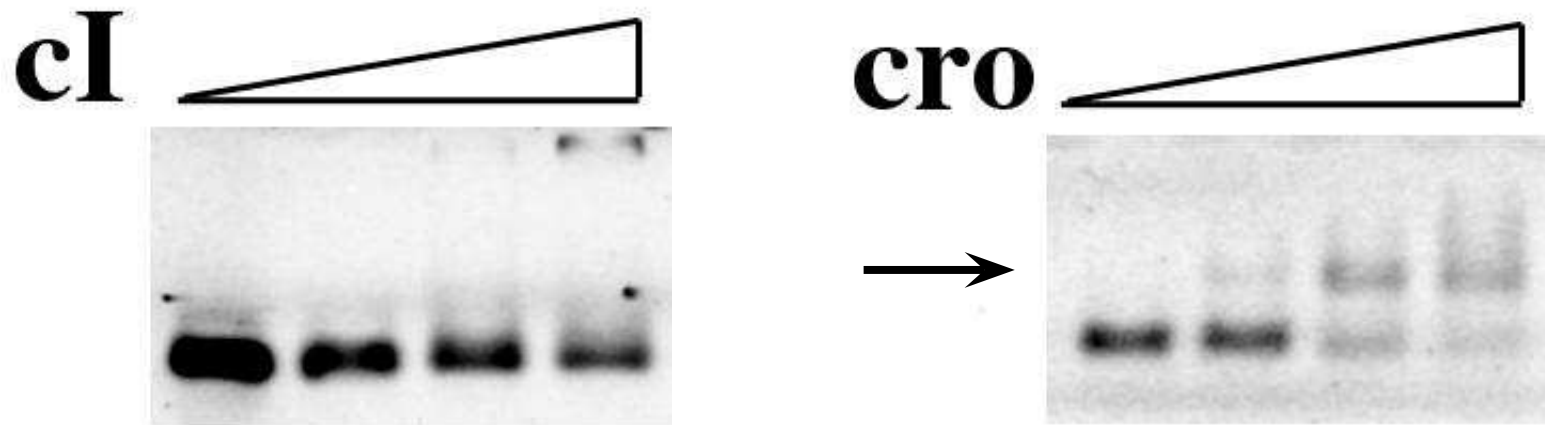
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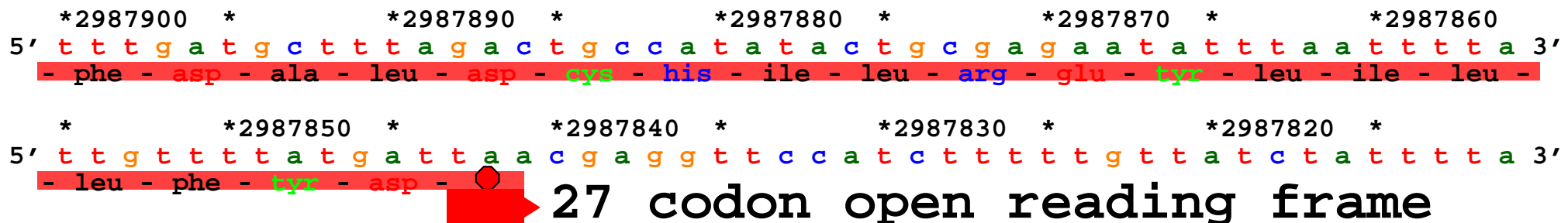
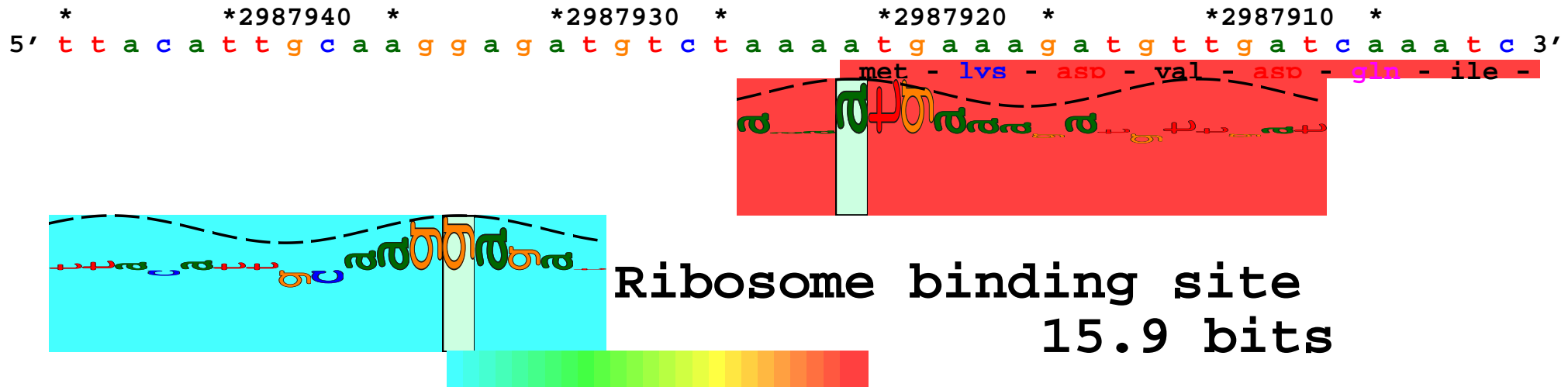


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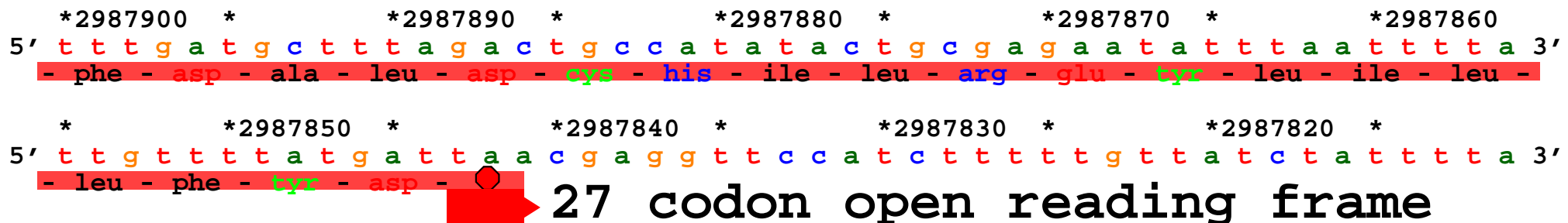
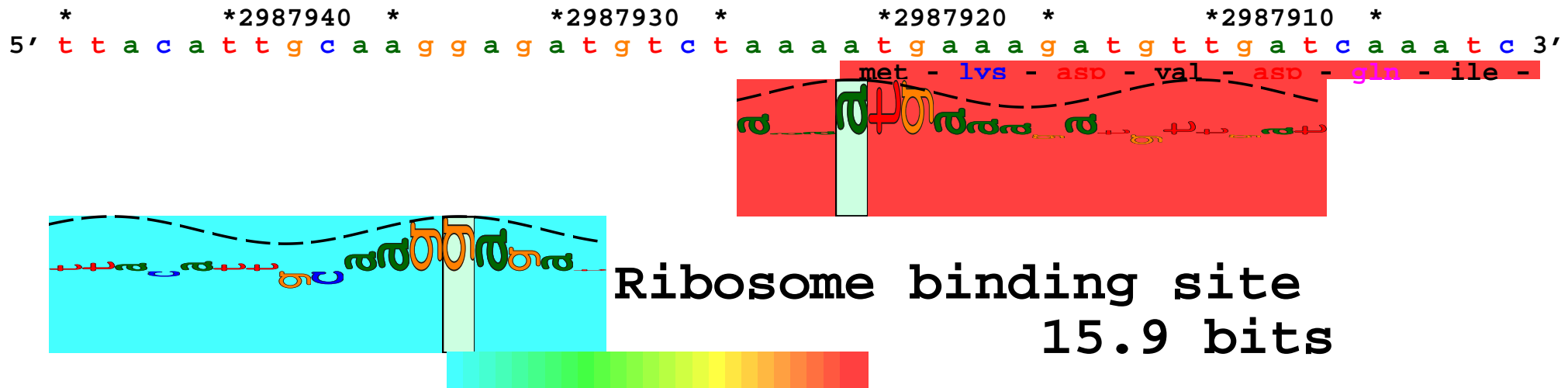
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Predicting Small Open Reading Frames in *E. coli*



Collaborators: Dr. Gisela Storz (NIH, NICHD, Bethesda, MD) and Dr. Kenneth Rudd (University of Miami School of Medicine, Miami, FL)

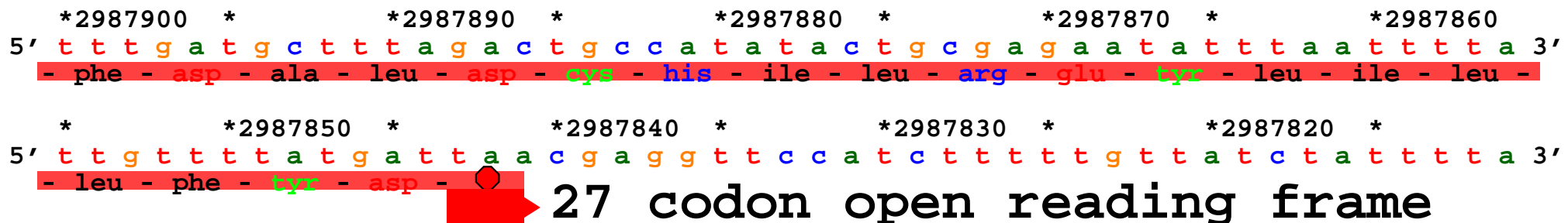
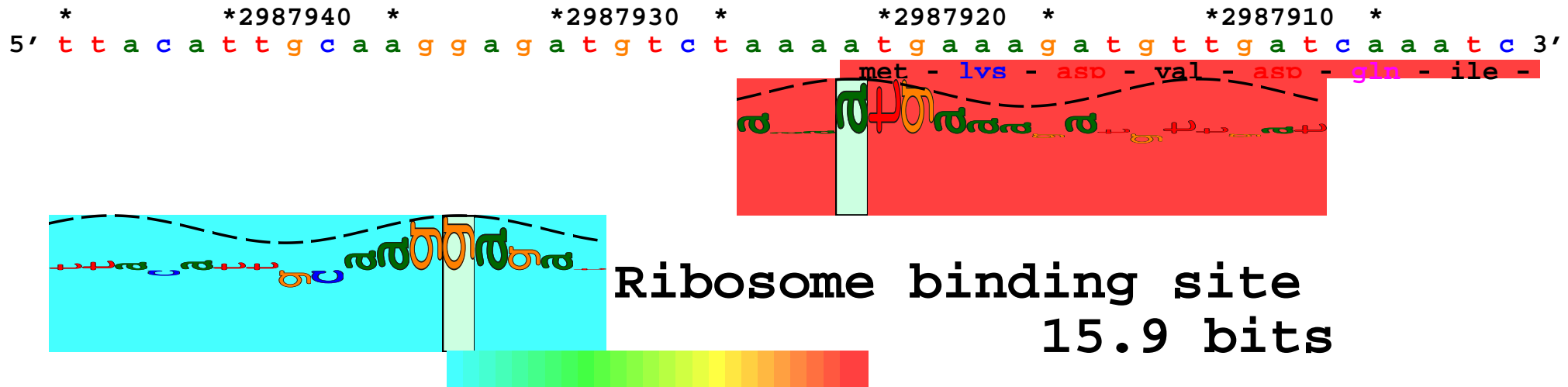
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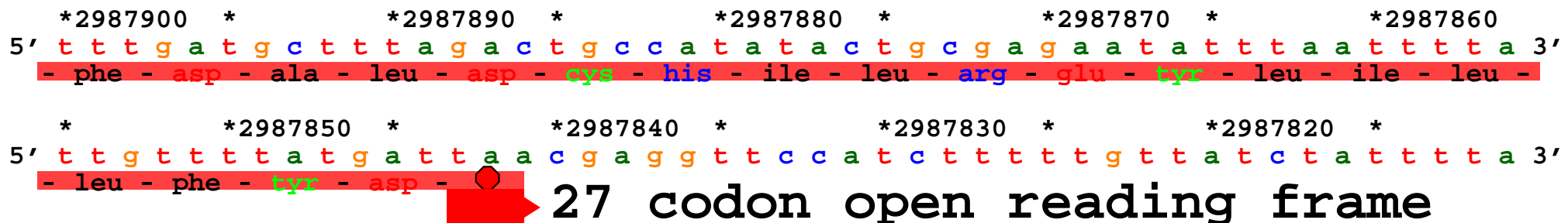
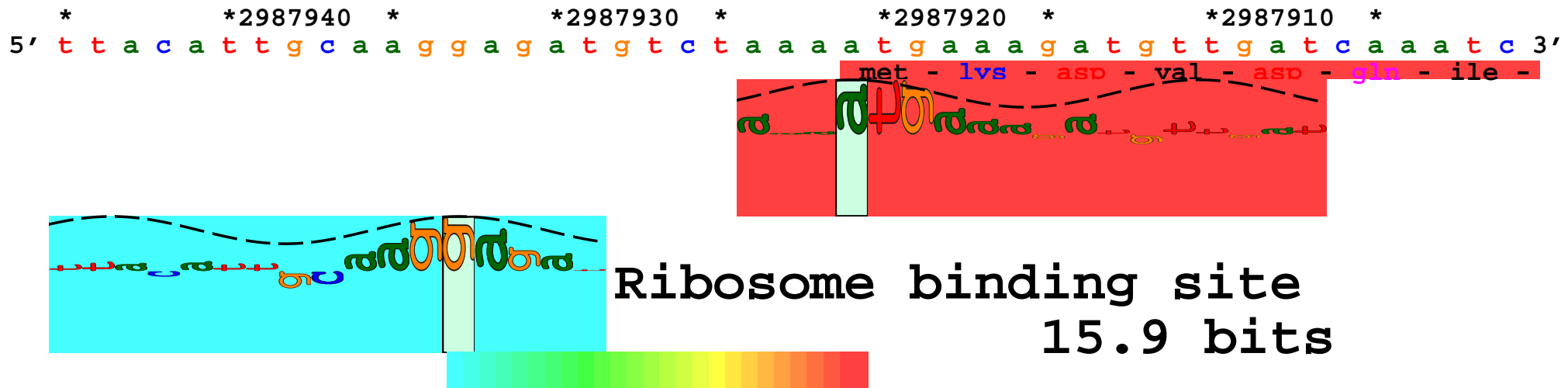
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