KNOTTED ELASTIC CURVES IN $\mathbb{R}^3$

JOEL LANGER AND DAVID A. SINGER

One of the oldest topics in the calculus of variations is the study of the elastic rod which, according to Daniel Bernoulli's idealization, minimizes total squared curvature among curves of the same length and first order boundary data. The classical term *elastica* refers to a curve in the plane or $\mathbb{R}^3$ which represents such a rod in equilibrium.

While the elastica and its generalizations have long been (and continue to be) of interest in the context of elasticity theory, the elastica as a purely geometrical entity seems to have been largely ignored.

Recently, however, Bryant and Griffiths [**1**, **3**] have found the elastica and its natural generalization to space forms (where arc length is generally not constrained) to be an interesting example in the context of the general theory of exterior differential systems. Independently, the present authors have studied 'free' elastic curves in space forms and have drawn connections to well-known problems in differential geometry [**4**, **5**].

From the geometric point of view, the *closed* elasticae and their global behaviour are naturally of particular interest. In the present paper we maintain this emphasis but return to the classical setting of Euclidean curves with fixed arc length. Specifically, we give a complete classification of closed elastic curves in $\mathbb{R}^n$ and determine the knottedness of these elasticae. We note that the integrability of the equations for a classical elastica was known already to Euler in the planar case and (essentially) to Radon in the case of $\mathbb{R}^3$ (see Blaschke's *Vorlesungen über Differentialgeometrie* I); to determine the *closed* elasticae, however, the chief problem is to understand the dependence of the resulting elliptic integrals on certain parameters.

Since the closed *planar* elasticae were well described already by Euler, and since uniqueness of solutions in the initial-value problem implies that any elastica in $\mathbb{R}^n$ must in fact lie in $\mathbb{R}^3$, it will suffice to present the following.

MAIN THEOREM. *There exists a countably infinite family of (similarity classes of) closed non-planar elastic curves in $\mathbb{R}^3$. All such elasticae are embedded and lie on embedded tori of revolution. Infinitely many of these are knotted and the knot types which thus occur are precisely the (m, n)-torus knots satisfying $m > 2n$. The integers m, n determine the elasticae uniquely (up to similarity).*

The reader may find it surprising that closed non-planar elasticae exist at all. Before presenting a proof of this fact, we explain heuristically why such elasticae should occur.

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Let $C(p)$ be the p-fold circle

$$x(s) = \frac{1}{p}\cos(ps), \qquad y(s) = \frac{1}{p}\sin(ps), \qquad 0 \leqslant s \leqslant 2\pi,$$

and let $C(-q)$ be the q-fold circle of opposite orientation

$$\bar{x}(s) = \frac{1}{q}\cos(-qs), \qquad \bar{y}(s) = \frac{1}{q}\sin(-qs).$$

Considered as curves in $\mathbb{R}^3$, $C(p)$ and $C(-q)$ are G-equivariantly regularly homotopic, where G is the group generated by rotation about the z-axis by an angle $(2\pi p/(p+q))$. Furthermore, $C(p)$ and $C(-q)$ locally minimize total squared curvature among G-symmetric curves of length 2π (as can be seen by second variation computations similar to those in [**4**]).

By a mountain-pass argument and the principle of symmetric criticality one may therefore expect a third critical point having the same symmetry. Since the only closed planar elasticae are k-fold circles and k-fold figure-eight curves (which possess a $\mathbb{Z}_2$ symmetry), the above third critical point will be non-planar except in the case when $p = q$ (where the p-covered figure eight is obtained).

We begin the proof by reviewing basic facts about elasticae in space forms; these are discussed in greater detail in [**4**]. In what follows G denotes the (constant) sectional curvature of the 3-manifold M.

The curvature k and torsion τ of an elastica γ in M satisfy the Euler equations

$$(1) \qquad 0 = 2k_{ss}+k^3-\lambda k-2k\tau^2+2kG, \qquad k^2\tau = c = \text{constant}.$$

Here λ is an undetermined constant which arises as a Lagrange multiplier.

The solutions to (1) are given in terms of three parameters α, p and w and the Jacobi elliptic functions $sn(x, p)$. Here α represents the maximum squared curvature, while p, w, with $0 \leqslant p \leqslant w \leqslant 1$, control the shape of the curve (this is elaborated upon in Figure 1). Note that the parameter w is equal to p/q [**4**].

The non-constant solutions of (1) are given by

$$(2) \qquad k^2 = u(s) = \alpha\left(1 - \frac{p^2}{w^2}\, sn^2(rs, p)\right), \qquad r = \frac{\sqrt{\alpha}}{2w},$$

$$(3) \qquad 2\lambda - 4G = \frac{\alpha}{w^2}(3w^2-p^2-1), \qquad 4c^2 = \frac{\alpha^3}{w^4}(1-w^2)(w^2-p^2).$$

From these equations one can see, for example, that a planar elastica (where $c = 0$) must satisfy either $w = 1$ or $w = p$. The first case yields curves with non-vanishing curvature ('orbitlike' elasticae) while the second yields curves whose curvature has alternating sign ('wavelike' elasticae). The condition that $p = 0$ gives rise to curves with constant curvature and torsion (helices). In all other cases τ is non-vanishing and non-constant.

Integration of (1) gives rise to the useful equation

$$(4) \qquad (k_s)^2+\tfrac{1}{4}k^4 + \left(G - \frac{\lambda}{2}\right)k^2 + \frac{c^2}{k^2} = \text{constant} = \frac{\alpha^2}{4} + \left(G - \frac{\lambda}{2}\right)\alpha + \frac{c^2}{\alpha}.$$

The following proposition is the key to integrating the Frenet equations for γ.

PROPOSITION 1. *A vector field J along a curve γ in M extends to a Killing field on M (more precisely, on the universal cover of M) if and only if J satisfies*

$$\text{(5)}\qquad \begin{cases} 0 = \langle \nabla_T J, T\rangle, \quad 0 = \langle (\nabla_T)^2 J + G\langle J, N\rangle N, N\rangle, \\ 0 = \left\langle (\nabla_T)^3 J - \dfrac{k_s}{k}(\nabla_T)^2 J + (G+k^2)\nabla_T J - \dfrac{k_s}{k} GJ, B\right\rangle, \end{cases}$$

where the vectors T, N, B form the Frenet frame for γ, and $(\nabla_T)^i$ denotes the i-th covariant derivative with respect to T.

We say that a vector field which satisfies (5) is *Killing along* γ. As observed in [4], if γ is an elastica, the vector field $J_0 = (k^2-\lambda)T + 2k_s N + 2k\tau B$ is always Killing along γ. We observe now that (1) and (5) imply that kB is also Killing along γ.

In the present case the Killing fields J_0 and kB are naturally related to a system of cylindrical coordinates (r, θ, z). For one thing, (4) implies that J_0 has constant magnitude $|J_0| = (\alpha-\lambda)^2 + 4c^2/\alpha$. Such a Killing field in $\mathbb{R}^3$ is obviously a translation (that is, a constant) field. Normalizing, we obtain one coordinate field $\partial/\partial z = J_0/|J_0|$.

Now kB has non-constant magnitude (unless γ is a line, a circle or a helix) while its dot product with J_0 is a constant, that is, $kB \cdot J_0 = 2c$. It follows that $J_1 = J_0 - (1/2c)|J_0|^2 kB$ is a rotation field perpendicular to J_0. Thus, for some normalization factor Q, the second coordinate field is given by $\partial/\partial\theta = QJ_1$. Finally, $\partial/\partial r$ is given in terms of a cross product

$$\frac{\partial}{\partial r} = \frac{J_0 \times B}{|J_0 \times B|}$$

(the correct sign for $\partial/\partial r$ is not obvious, but follows from computations similar to those given below).

PROPOSITION 2. *Let (r, θ, z) be cylindrical coordinates whose coordinate fields are $\partial/\partial r$, $\partial/\partial\theta$, $\partial/\partial z$ given above, and let $(r(s), \theta(s), z(s)) = \gamma(s)$. Then*

$$r_s = \frac{u_s}{(|J_0|^2 u - 4c^2)^{1/2}}, \qquad z_s = \frac{u-\lambda}{|J_0|} = \frac{\alpha}{|J_0|}\left(1 - \frac{p^2}{w^2} sn^2(rs, p)\right) - \frac{\lambda}{|J_0|}$$

and

$$\theta_s = \frac{c|J_0|(u-\lambda)}{|J_0|^2 u - 4c^2} = \frac{c}{|J_0|}\left[1 + \left(\frac{4c^2 - \lambda|J_0|^2}{\alpha(\alpha-\lambda)^2}\right)\cdot\left(\frac{1}{1-\beta^2 sn^2(rs, p)}\right)\right],$$

where $\beta^2 = |J_0|^2 p^2/(\alpha-\lambda)^2 w^2$.

Proof. Writing $T = r_s \dfrac{\partial}{\partial r} + \theta_s \dfrac{\partial}{\partial\theta} + z_s \dfrac{\partial}{\partial z}$ and taking dot products with the above formulas for $\partial/\partial r$, $\partial/\partial z$, one can easily obtain r_s, z_s.

To get θ_s one must first determine the normalization factor Q. This can be done by choosing Q such that QJ_1 has the proper length at maxima of $k(s)$, that is, at maxima of $r(s)$; in fact, the length of $\partial/\partial\theta$ at such a point $\gamma(s_0)$ is the reciprocal of the curvature k_c of the circle $r = r(s_0)$, $z = z(s_0)$.

Now k_c can be computed as follows. At maxima of k the unit tangent vector T has vertical component $\frac{T \cdot J_0}{|J_0|} = \frac{\alpha - \lambda}{|J_0|}$, and hence horizontal component $\frac{2c}{\sqrt{\alpha}|J_0|}$. Thus

$$k_c = \frac{\sqrt{\alpha}|J_0|}{2c}\left|\nabla_T\left(\frac{\partial}{\partial\theta}\Big/\left|\frac{\partial}{\partial\theta}\right|\right)\right| = \frac{\sqrt{\alpha}|J_0|}{2c}|\nabla_T(J_1/|J_1|)|\,.$$

Since $T(|J_1|) = 0$, $k_s = 0$ and $|J_1| = \sqrt{\alpha}(\alpha-\lambda)/2c|J_0|$ at such points, one easily computes

$$k_c = \frac{|\nabla_T J_1|}{\alpha - \lambda} = \frac{|J_0|^2}{2\sqrt{\alpha}(\alpha-\lambda)} = \frac{|J_0|^3}{4c|J_1|},$$

concluding that $Q = 4c/|J_0|^3$.

Straightforward substitution into $T \cdot \frac{\partial}{\partial\theta}\Big/\left|\frac{\partial}{\partial\theta}\right|^2$ now gives the desired expression for θ_s (the final expression is not the simplest, but it is the most convenient for integration).

By virtue of Proposition 2 we can already assemble the following partial description of γ (we neglect the special case when k = constant). The entire elastica γ lies between two concentric cylinders (the inner cylinder possibly degenerating to a line). The critical points and periodicity of $k(s)$ and $r(s)$ coincide, the two functions passing in one period from a minimum (on the inner cylinder) to a maximum (on the outer cylinder) and back to a minimum, with no other critical points. Meanwhile, the critical points of $z(s)$ and $\theta(s)$ coincide, each having zero, one, or two critical points in each period of $k(s)$; also, $z(s)$, $\theta(s)$ themselves differ from periodic functions by linear functions. Finally, we note that we have already computed the radius of the outer cylinder $R_c = 1/k_c = 2\sqrt{\alpha}(\alpha-\lambda)/|J_0|^2$ and can get the radius of the inner cylinder by integrating r_s and subtracting from R_c:

$$(6) \qquad r_c = R_c - \left[\frac{2(|J_0|^2 u - 4c^2)^{1/2}}{|J_0|^2}\right]_{u_{\min}}^{\alpha} = \frac{\alpha^{3/2}}{|J_0|^2 w^3}(1-p^2-w^2)(w^2-p^2)^{1/2}\,.$$

Now let Δz and $\Delta\theta$ denote the net changes in $z(s)$, $\theta(s)$, respectively, through one period of k. Then γ closes up if and only if $\Delta z = 0$ and $\Delta\theta$ is rationally related to 2π.

Our goal now is to investigate these two conditions, beginning with the much simpler $\Delta z = 0$. In our computations of $\Delta z, \Delta\theta$, we shall be using the standard notation K, E, Λ_0 to denote the complete elliptic integrals of the first and second kinds and the Heuman lambda function, respectively (we refer the reader to **[2]** for computational facts about elliptic functions).

Since $k(s)$ has period $2K/r$ and since k is even about $s = K/r$ we have

$$\Delta z = 2\int_0^{K/r} \frac{u-\lambda}{|J_0|}\,ds = \frac{2}{r|J_0|}\int_0^K \left((\alpha-\lambda) - \alpha\frac{p^2}{w^2}sn^2(x,p)\right)dx$$

$$= \frac{2}{r|J_0|}\left[(\alpha-\lambda)K - \frac{\alpha}{w^2}(K-E)\right].$$

Using (3) one obtains that $(\alpha-\lambda) = (\alpha/2w^2)(1+p^2-w^2)$, and so the condition that $\Delta z = 0$ is the transcendental equation

$$1+w^2-p^2-2\frac{E}{K} = 0. \tag{7}$$

For elasticae satisfying (7) we can enhance the description. First, comparison of (6) and (7) shows that $r_c > 0$ (unless γ is planar). Secondly, $z(s)$ is also periodic in this case, so the fact that $z(s)$ and $r(s)$ have only two critical points in each period implies that γ is embedded and lies on an embedded torus of revolution.

To determine the nature of the set of closed elasticae it remains only to investigate the behaviour of $\Delta\theta$ for those elasticae satisfying (7). For this, as well as for understanding the set of *all* elasticae, it is helpful to fix the maximum curvature $\sqrt{\alpha}$ and consider the (p^2, w^2)-parameter space of (similarity classes of) elasticae pictured in Figure 1.

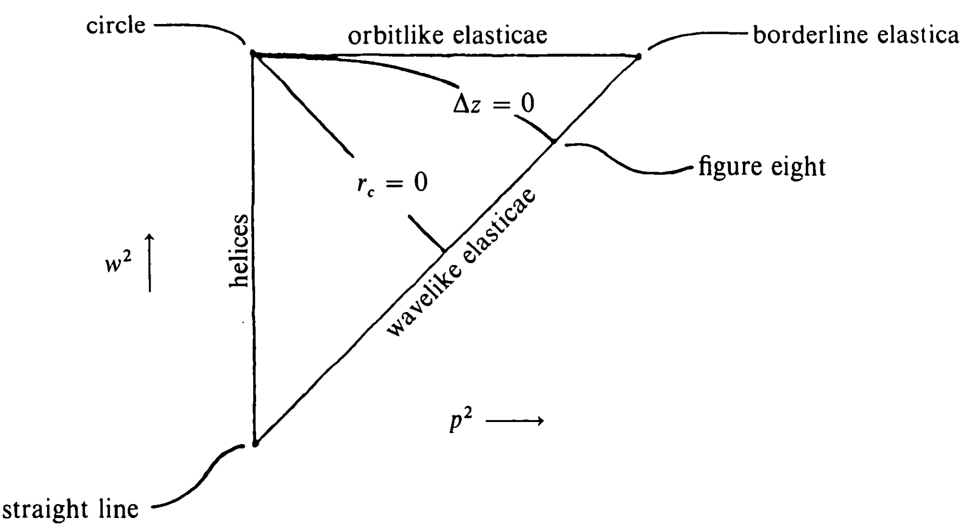

FIG. 1

In Figure 1 the planar elasticae are represented by the points on two sides of the triangle, helices on the third side, and the other non-planar elasticae by points in the interior. The line $r_c = 0$ represents the non-planar elasticae which pass through the z-axis, and the curve $\Delta z = 0$ represents the set of solutions of (7). The upper right-hand corner $w = p = 1$, labelled 'borderline elastica' represents the special (planar) elastica having curvature $k = \sqrt{\alpha}\,\mathrm{sech}\,(rs)$.

To study the behaviour of $\Delta\theta$ on this triangle we now express $\Delta\theta$ as a function of the parameters p^2, w^2. Unfortunately, this will lead to increasingly complicated expressions in p^2, w^2; thus, for the sake of economy, we shall refer to the following substitutions:

$$\begin{cases} U = \sqrt{1-p^2}, \quad V = \sqrt{1-w^2}, \quad X = \sqrt{w^2-p^2}, \quad Y = \sqrt{1+p^2-w^2}, \\ Z = \sqrt{Y^4+4V^2X^2}, \quad A = \sqrt{\beta^2-p^2}, \quad B = \sqrt{1-\beta^2}, \quad \Omega = \dfrac{2VXZ}{wY^2}. \end{cases} \tag{8}$$

PROPOSITION 3.

$$(9)\qquad \begin{cases} \Delta\theta = \Omega K(p) \pm \pi[1-\Lambda_0(\phi,p)], \\ \sin\phi = \dfrac{B}{U} = \dfrac{X}{U}\cdot\dfrac{|1-w^2-p^2|}{wY^2}. \end{cases}$$

In the formula for $\Delta\theta$ the $\pm$ sign agrees with the sign of $1-w^2-p^2$; so $+$ holds below the line $r_c = 0$, while $-$ holds above $r_c = 0$ (and on $r_c = 0$ the value of $\Delta\theta$ is only defined mod 2π).

Proof. The derivation of the above formula for $\Delta\theta$ is tedious but can be organized as follows. One begins by collecting some expressions familiar from above:

$$|J_0|^2 = (\alpha-\lambda)^2\frac{4c^2}{\alpha},\quad (\alpha-\lambda) = \frac{\alpha}{2w^2}Y^2,\quad \lambda = \frac{\alpha}{2w^2}(3w^2-p^2-1),\quad \frac{4c^2}{\alpha} = \frac{\alpha^2}{w^4}V^2X^2.$$

Substitutions and simplifications lead to

$$\beta^2 = \frac{|J_0|^2p^2}{(\alpha-\lambda)^2w^2} = \frac{p^2Z^2}{w^2Y^4},\qquad B^2 = \frac{X^2(1-w^2-p^2)^2}{w^2Y^4},\qquad A^2 = \frac{p^2V^2(1+w^2-p^2)^2}{w^2Y^4}.$$

Using these formulas one eventually obtains

$$(10)\qquad 4c^2-\lambda|J_0|^2 = \frac{(\alpha-\lambda)^3ABw^2}{pVX}.$$

The above formulas for $B^2 = 1-\beta^2$ and $A^2 = \beta^2-p^2$ imply that $p^2 \leqslant \beta^2 \leqslant 1$, so formula 412.01 of [2] gives

$$\int_0^K \frac{dx}{1-\beta^2 sn^2 x} = K + \frac{\pi\beta[1-\Lambda_0(\phi,p)]}{2AB},$$

with ϕ as in (9).

With $x = rs = \dfrac{\sqrt{\alpha}}{2w}s$, integration of θ_s and simplification of the K coefficient leads to

$$\Delta\theta = 2\int_0^{K/r}\theta_s ds$$

$$= \frac{4w}{\sqrt{\alpha}}\cdot\frac{c}{|J_0|}\cdot\frac{1}{\alpha(\alpha-\lambda)^2}\left[(\alpha-\lambda)|J_0|^2K + \frac{4c^2-\lambda|J_0|^2}{2AB}\pi\beta[1-\Lambda_0(\phi,p)]\right].$$

Finally, (10) yields a remarkable cancellation and the desired formula.

It is now a simple matter to read off the behaviour of $\Delta\theta$ on the boundary of the parameter triangle. On the left and upper boundaries $\sin\phi = 1$ so that $\Lambda_0 = 1$, and on the diagonal boundary $\sin\phi = 0$, so that $\Lambda_0 = 0$. The formula for $\Delta\theta$ now gives the values indicated in Figure 2.

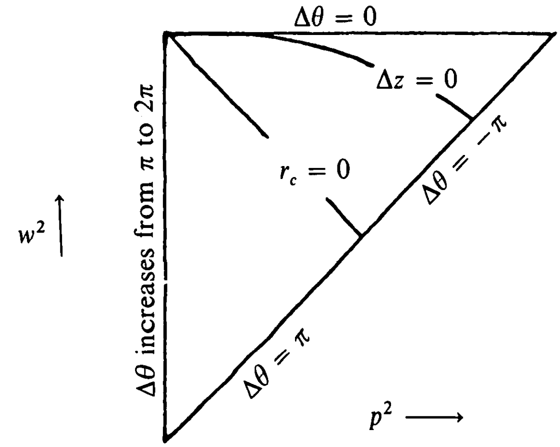

FIG. 2

Note that $\Delta\theta$ jumps by 2π at the line $r_c = 0$ where elasticae cross through the z-axis, and by π at the borderline elastica. Otherwise $\Delta\theta$ is continuous. Most significant for our present purpose is the fact that, along the curve $\Delta z = 0$, $\Delta\theta$ varies continuously from 0 to $-\pi$ and hence achieves all intermediate values (the value of $\lim_{p \to 0, \Delta z = 0} \Delta\theta \to 0$ is only suggested by Figure 2, but is easily obtained from (9) once one observes that along the curve $\Delta z = 0$ the quantity V^2 behaves like p^4 for p close to 0).

This proves the existence of all the closed elasticae described in the main theorem. To show that there are no others requires that we establish monotonicity of $\Delta\theta$ along the curve $\Delta z = 0$. We accomplish this by proving that

$$\frac{\partial}{\partial p}\Delta\theta + \left(\frac{\partial}{\partial w}\Delta\theta\right)\cdot\frac{dw}{dp} < 0$$

for $w(p)$ defined by (7).

The computations are quite formidable, but suitable strategy (as outlined in the appendix) leads to massive cancellations and ultimately produces relatively simple partial derivative formulas:

$$\frac{\partial}{\partial p}\Delta\theta = \frac{2wV}{pU^2XZ}\left[(1+3w^2-5p^2)E+(1-p^2)(3p^2-1-3w^2)K\right],$$

$$\frac{\partial}{\partial w}\Delta\theta = \frac{2}{XVZ}\left[(1-p^2+3w^2(p^2-w^2))K+(3w^2-1-p^2)E\right].$$

Along the curve $\Delta z = 0$, that is, for $w(p)$ defined by (7), the bracketed terms above can be rewritten

$$[(1+3w^2-5p^2)E+(1-p^2)(3p^2-1-3w^2)K]$$

$$= \frac{2}{K}[(E-K)(E-(1-p^2)K)+E(2E-(2-p^2)K)]$$

$$= -\big[(1-p^2+3w^2(p^2-w^2))K+(3w^2-1-p^2)E\big].$$

But $(E-K) < 0$, $\big(E-(1-p^2)K\big) > 0$ and $\big(2E-(2-p^2)K\big) < 0$ (the last two of these can be verified by differentiation), and differentiation of (7) gives $dw/dp = -[E-(1-p^2)K]^2/wK^2p(1-p^2) < 0$, and so the total derivative of $\Delta\theta$ along the curve $\Delta z = 0$ is negative as required.

We conclude with some speculation on the question of stability. Recalling the minimax argument given earlier we observe that the elasticae obtained by that argument are in natural one-to-one correspondence with the elasticae of the main theorem; specifically, n is the smaller of p, q and $m = p+q$. Since the minimax critical points are unstable, we are led to the following.

CONJECTURE. The circle is the only stable closed elastica in $\mathbb{R}^3$.

In particular, this implies that a knotted wire cannot rest in stable equilibrium without points of self-contact. One might ask what actually happens when a knot is formed in a piece of springy wire (the ends joined together smoothly). Experiments produce some very 'canonical'-looking space curves with impressive symmetry (for example, the figure-eight knot or the Chinese button knot). Invariably, points of self-contact are indeed observed. In fact, it is tempting to conjecture that there must be at least three such points (counting multiplicity) in a knotted wire.

APPENDIX

The key to successfully differentiating $\Delta\theta$ lies in combining terms in the correct order. Failure to do this leads to the creation of high-degree polynomials, which ultimately can be factored but not easily. We illustrate this by computing $\partial\Delta\theta/\partial p$; the other partial derivative is computed in a similar fashion.

Observing that $1-U^2\sin^2\phi = \beta^2$, we have

$$\frac{\partial}{\partial p}\Delta\theta = K\Omega\frac{\partial}{\partial p}\log K\Omega + \pi\frac{\partial}{\partial p}\Lambda_0(\phi, p)$$

$$= K\Omega\left[\frac{-p}{X^2} - \frac{2p}{Y^2} + \frac{2pY^2}{Z^2} - \frac{4pV^2}{Z^2} + \frac{E}{pU^2K} - \frac{1}{p}\right]$$

$$+ \left[\frac{2(E-K)\sin\phi\cos\phi}{p\beta}\right] + \left[\frac{2(E-U^2\sin^2\phi K)}{\beta}\right]\frac{\partial\phi}{\partial p}.$$

From formula (9) one can compute the derivative of ϕ, yielding

$\dfrac{\partial\phi}{\partial p} = \dfrac{V(1+w^2-3p^2)}{XU^2Y^2}$. The coefficient of E is the sum of three terms:

$$\frac{2VX\beta}{U^2p^2} + \frac{2VX(1-p^2+w^2)(p^2+w^2-1)}{\beta w^2Y^4U^2} + \frac{2V(1+w^2-3p^2)}{\beta XU^2Y^2}.$$

The first two terms combine to yield $4XV/\beta U^2Y^2$. When combined with the third term, this gives the coefficient of E as

$$\frac{2V(1+3w^2-5p^2)}{\beta U^2Y^2X} = \frac{2wV(1+3w^2-5p^2)}{pU^2XZ}.$$

The coefficient of K is the sum of seven terms. The first, second, and fifth terms sum to

$$\frac{2V\beta(2p^4-w^2(1+3p^2)+w^4)}{p^2XY^2}.$$

The third, fourth, sixth and seventh terms combine to give

$$\frac{-4p^2VX(p^2+w^2-1)(1-p^2+w^2)}{\beta w^2Y^6}.$$

These two terms add up to $2wV(3p^2-3w^2-1)/pXZ$. Thus we have

$$\frac{\partial}{\partial p}\Delta\theta = \frac{2wV}{pU^2XZ}[(3p^2-3w^2-1)(1-p^2)K+(1+3w^2-5p^2)E].$$

References

1. R. Bryant and P. Griffiths, 'Reduction of order for constrained variational problems, and $\int k^2/2\,ds$', *Amer. J. Math.*, to appear.
2. P. F. Byrd and M. D. Friedman, *Handbook of elliptic integrals for engineers and physicists* (Springer, Berlin, 1954).
3. P. Griffiths, *Exterior differential systems and the calculus of variations*, Progress in Mathematics 25 (Birkhauser, Boston, 1982).
4. J. Langer and D. Singer, 'On the total squared curvature of closed curves', *J. Differential Geom.*, to appear.
5. J. Langer and D. Singer, 'Curves in the hyperbolic plane and mean curvature of tori in 3-space', *Bull. London Math. Soc.*, 16 (1984), 531–534.

Department of Mathematics,
Case Western Reserve University,
Cleveland,
Ohio 44106,
U.S.A.

Current address of first author:
Max Planck Institut für Mathematik,
Gottfried-Claren-Strasse 26,
5300 Bonn 3,
West Germany.