MATH 424 – Assignment #5 – Extra Problem

Due Monday, March 15, 2021

A. Let $p \in (0,1)$ and let $\ell_p := \{x = (x_j)_{j=1}^{\infty} : \sum_{j=1}^{\infty} |x_j|^p < \infty\}$. Show that
(i) $\rho(x,y) := \sum_{j=1}^{\infty} |x_j - y_j|^p$ is a metric on ℓ_p
(ii) The metric space (ℓ_p, ρ) is complete.