MATH 424 – SPRING 2021 – Assignment # 2 comments

Exercise 5.7a[1], p.155

For scalars, this says that if $|1-t| < 1$, and hence $t \neq 0$, then $\sum_{n=0}^{\infty}(1-t)^n = 1/t$ (or, equivalently, $t\sum_{n=0}^{\infty}(1-t)^n = 1$, which follows readily from the formula for the sum of a geometric series. So one obvious strategy is to try to come up with a proof of one of these formulae which carries over to the operator setting. (In particular, one has to pay attention to possible non-commutativity.) Also, it may be convenient to substitute $s = 1 - t$.

Exercise 5.12b

It may be more geometrically appealing to replace $\|x + \mathcal{M}\|$ by $\|x - \mathcal{M}\| = \mathrm{dist}(x, \mathcal{M})$, which is allowed since $\mathcal{M}$ is a vector subspace. First think about the Euclidean setting, in which case actually there is x with $\|x\| = 1$ and $\mathrm{dist}(x, \mathcal{M}) = 1$. Next, suppose $\dim(\mathcal{X}) < \infty$; the previous statement still holds, but is less obvious. Finally, consider the general case.

Exercise 5.17

It is not absolutely necessary to appeal to Exercise 12b, but if you do, use it with $\mathcal{M} = f^{-1}(0)$. Either way, you may prove and use the fact that if a linear functional f is *not bounded*, then $f(\overset{\circ}{B}(x,r)) = \mathbb{K}$ for any ball $\overset{\circ}{B}(x,r)$ (i.e., open ball centered at x with radius $r > 0$). Here $\mathbb{K} = \mathbb{R}$ or $\mathbb{K} = \mathbb{C}$ depending whether the space $\mathcal{X}$ is real or complex.

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[1] Exercises being numbered by chapter, this is Exercise 7 in Chapter 5.