April 29, 2018

Additional (way–post–publication) corrigenda for

[**DS**] K. R. Davidson and S. J. Szarek, *Local operator theory, random matrices and Banach spaces.* In "Handbook on the Geometry of Banach spaces," Volume 1, W. B. Johnson, J. Lindenstrauss eds., Elsevier Science 2001, pp. 317-366. Addenda and Corrigenda, in Volume 2, 2003, pp. 1819-1820.

• Here is the correct statement of Problem 1.17 from [**DS**] (p. 333).

PROBLEM 1.17. *Given* $\varepsilon > 0$, *is there a constant* $C(\varepsilon)$ *independent of* n *such that, for every* $n \times n$ *matrix* T *with* $\|T\| \le 1$, *there exists a diagonal operator* D *and an invertible operator* W *such that*

$$\|T - WDW^{-1}\| \le \varepsilon \quad \text{and} \quad \|W\| \, \|W^{-1}\| \le C(\varepsilon)?$$

A compactness argument shows that, for each fixed dimension n, there is a constant $C(\varepsilon, n)$ which works. The reader who would like to get a feel for this question may consider the case when T is a Jordan cell.

Added October 10, 2019 : As indicated in [**DS**], Problem 1.17 was posed in [**DHS**]. It was shown in [**BKMS**] that, in the complex case, $C(\varepsilon, n) = 4n^{3/2}(1 + 1/\varepsilon)$ works. A weaker explicit estimate can be found in [**D**], where a version of the problem is also stated.

[**BKMS**] J. Banks, A. Kulkarni, S. Mukherjee, N. Srivastava, *Gaussian Regularization of the Pseudospectrum and Davies' Conjecture*, arXiv:1906.11819

[**DHS**] K.R. Davidson, D.A. Herrero and N. Salinas, *Quasidiagonal operators, approximation, and C*-algebras*, Indiana Univ. J. Math. **38** (1989), 973-998.

[**D**] E. Brian Davies. *Approximate diagonalization.* SIAM J. Matrix Anal. Appl. **29**(4):1051-1064, 2007.

• Here is the correct statement of Theorem 2.4. from [**DS**] (p. 345).

THEOREM 2.4. *Let* $G = G^{(n)}$ *be an* $n \times n$ *random matrix whose entries are independent identically distributed Gaussian random variables following the* $N(0, 1/n)$ *law. and let* $s_1 \ge s_2 \ge \ldots \ge s_n$ *be the singular values of* G. *Then*

$$\mathcal{P}\Big(s_{n-d+1} \ge \beta \frac{d}{n}\Big) \le \exp\left(-c\beta^2 dd'\right), \quad \mathcal{P}\Big(s_{n-d+1} \le \alpha \frac{d}{n}\Big) \le (C\alpha)^{d^2}$$

for $1 \le d \le n$, $\beta \ge \beta_0$, *and* $\alpha \ge 0$, *where* $d' = \min\{d, n-d\}$. *Above,* c, C *and* β_0 *are universal positive constants. Apart from the precise values of these constants, the estimates are optimal.*

The references for the above are Theorem 1.2 and Lemma 3.1 in Ref. [**S**] below, the latter covering the first bound in the range $d > n/2$. (As originally stated, the first bound was valid only in the $d \le n/2$ range; we thank Martin Kroll for bringing this issue to our attention.) Also, the caveat "except possibly (..) when $n - d = o(n)$" is not really needed: while the fluctuations of the largest singular values are more precisely described by the Tracy-Widom phenomenon exemplified in Theorem 2.8 of [**DS**], this phenomenon belongs to the "small deviation" regime: it shows up only for $\beta < \beta_0$ and not in the "large deviation" setting of Theorem 2.4. For a selection of related ready-to-use bounds with reasonable constants the reader is referred to Chapter 6 of the recent book [**AS**].

[**AS**] G. Aubrun and S. J. Szarek, *Alice and Bob Meet Banach. The Interface of Asymptotic Geometric Analysis and Quantum Information Theory.* Mathematical Surveys and Monographs, Vol. 223, Amer. Math. Soc. 2017, 414 + xxi pp.

[**S**] S. J. Szarek, *Condition numbers of random matrices*, J. Complexity **7** (1991), 131-149.