

### On the Convexified Sauer-Shelah Theorem

*Abstract:* Let  $A$  be a subset of  $\{0, 1\}^n$ . Given  $\epsilon > 0$ , we can find a subset  $I$  of  $\{1, \dots, n\}$  such that the convex hull in  $\mathbf{R}^I$  of the projection of  $A$  onto  $\{0, 1\}^I$  contains the cube  $[1/2 - \epsilon, 1/2 + \epsilon]^I$ , and that  $\#I \geq n - K(n\epsilon + \sqrt{n \log \frac{2^n}{\#A}})$ , where  $K > 0$  is a universal constant.