Computational Neuroscience

aka MATH 378/478, BIOL 378/478, EECS 478, NEUR 478, EBME 478, COGS 378

Homework Problems # 1

due Tuesday 2/5/08 in class

1. * Izhikevich 2.1
2. * Refering to Izh. problem 2.1: Assuming that the internal and external ion concentrations remain the same, how would the resting potentials of each ion change if the temperature were changed from 20 degrees (lab temperature) to 37 degrees Celsius (body temperature)?
3. * Izh. 2.2
4. * Izh. 2.3
5. This problem refers to the first "hands-on" exercise in the online NEURON course materials.
   (a) * Run the file "bilayer.hoc" in NEURON. Activate the "pas" mechanism (passive conductances) and the "hh" mechanism (Hodgkin-Huxley conductances). Under RunControl, set the initial voltage to -70 mV and hit "Init". Use the command line window to find the values of the potassium and sodium potentials and conductances (soma.ek, soma.ena, soma.gna_hh and soma.gk_hh) used in the simulation for this value of the voltage.
   (b) * Calculate

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   (soma.gna_hh*soma.ena + soma.gk_hh*soma.ek)/(gna_hh + gk_hh)
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   Under PointProcessManager, select Show/Parameters. Turn off the current injection (set amp to 0 nA). Under RunControl, change Tstop to 500 ms. Init & run to find the steady-state voltage. Recalculate

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   (soma.gna_hh*soma.ena + soma.gk_hh*soma.ek)/(gna_hh + gk_hh)
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   at the steady state voltage. Does this quantity match the steady-state voltage? (See next question.)

(c) ** Explain why the steady-state voltage and the weighted average of reversal potentials calculated above don't agree, and find which combination of conductances and reversal potentials within the NEURON simulation *does* account for the steady state resting potential for the single compartment bilayer model.

6. ** Consider a semipermeable membrane (no ion channels, just an average bulk permeability) of thickness L with ion concentration equal to c_{in} on one side (at $x = 0$) and c_{out} on the other (at $x = L$). Suppose that at steady state the electric field is constant throughout the membrane, so that the voltage changes linearly by $V = \phi(0) - \phi(L)$ from inside to out. Let D be the diffusion constant of the ion in the membrane, z be the ionic charge, F Faraday's constant, T temperature, R Boltzmann's constant and J the flux of ions per second per unit area crossing the membrane (an unknown, but constant at steady state). Under these assumptions the concentration obeys a form of the Nernst-Planck equation,

$$J = -D\frac{dc}{dx} + c\frac{zFVD}{RTL}$$

The first term on the RHS represents the contribution to the flux made by passive diffusion, while the second term represents the contribution due to the electrical potential.

Solve the Nernst-Planck equation for $c(x)$ and J to obtain the Goldman-Hodgkin-Katz current equation for the ionic current I

$$I = P\frac{z^2F^2}{RT}V\frac{c_{in} - c_{out}\exp[\frac{-zFV}{RT}]}{1 - \exp[\frac{-zFV}{RT}]}$$

where $P = D/L$ is the so-called *permeability* of the membrane to this particular ion.

7. ** Show that $I = 0$ when V is equal to the Nernst potential for the ion.

8. ** For multiple currents from multiple ions (say, Na^+, K^+ and Cl^-), the steady state condition becomes

$$0 = \sum_{z=1} P_j\frac{c_{in,j} - c_{out,j}\exp[\frac{-FV}{RT}]}{1 - \exp[\frac{-FV}{RT}]} + \sum_{z=-1} P_j\frac{c_{in,j} - c_{out,j}\exp[\frac{FV}{RT}]}{1 - \exp[\frac{FV}{RT}]}.$$

Solve this equation for V to obtain the Goldman-Hodkin-Katz potential equation

$$V_r = -\frac{RT}{F}\log\left[\frac{P_{Na}[Na^+]_{in} + P_K[K^+]_{in} + P_{Cl}[Cl^+]_{out}}{P_{Na}[Na^+]_{out} + P_K[K^+]_{out} + P_{Cl}[Cl^+]_{in}}\right]$$

9. *** Extend the GHK potential equation to the case of a monovalent (e.g. K^+) and a divalent (e.g. Ca^{2+}) ion.

* denotes problems to be done by all students.

** denotes problems to be done students enrolled in MATH 378 or any of the cross-lists numbered 478. Any math or statistics majors and any graduate students enrolled in any section should do these problems as well.

*** denotes problems to be done by students enrolled under MATH 478, or math graduate students enrolled under any course number.