Computational Neuroscience

aka MATH 378/478, BIOL 378/478, EECS 478, NEUR 478, EBME 478, COGS 378

Homework Problems # 3

due Tuesday 4/8/08 in class

1. ** Consider the Hodgkin-Huxley model defined in section 2.3.1 of Izhikevich. Prove that if m, n and h have values in the open interval $0 < m, n, h < 1$ at $t = 0$, then regardless of the initial voltage, the functions $m(t), n(t)$ and $h(t)$ never leave the interval $[0, 1]$.

2. Download the file single-cell-example.hoc from the course website. This file, reproduced from The NEURON Book (Hines and Carnevale) uses hoc code to create a single cell with a few compartments (soma, apical dendrite, basal dendrite, axon) and provides it with an alpha synapse at the middle of the soma. Once you load the hoc file you can run the model in NEURON by typing `go()` at the `oc>` prompt. The graph of voltage versus time should show an action potential caused by the excitatory synaptic input.

   (a) * Change the synapse's peak conductance by varying `syn.gmax` (at the `oc>` prompt, type `syn.gmax=.04` etc.) Find the threshold for generating an action potential to two significant digits.

   (b) * Using the NEURON Main Menu / Tools / Point Processes / Managers / Point Manager, set up a second AlphaSynapse with onset = 0.5 ms, tau = 0.1 ms, gmax = 0.04 μS, and e = -70mV. Set the first synapse's peak conductance to zero (`oc> syn.gmax=0`) to see what the effect of the new synapse is. Restore `syn.gmax` to 0.05 and run the simulation with both synapses in place. Then vary the peak conductance of the second synapse: try setting `AlphaSynapse[1].tau` to 0.3 ms, 1 ms, and 3 ms. Describe the effect the second synapse has on the firing of the cell, and explain it.

3. * Use XPP/AUTO or another system (e.g. Matlab's numerical continuation package MATCONT) to create a bifurcation diagram and superimposed trajectory for the elliptic burster, written in normal form as:

$$\begin{aligned} u' &= u(\lambda + R - R^2) - v + c \\ v' &= v(\lambda + R - R^2) + u + c \end{aligned}$$

$$\begin{aligned}\lambda' &= \epsilon(R_0 - R) \\ R &= u^2 + v^2\end{aligned}$$

with parameters $\epsilon = 0.01, R_0 = 0.3, c = 0.01$. Let time run from 0 to 1000. See the instructions on pages 193-4 of Simulating, Analyzing, and Animating Dynamical Systems by Ermentrout (Exercise 7.5.1 of this book).

4. Download the NEURON code for Mainen and Sejnowski's 1996 paper on the influence of dendritic structure on firing patterns of cortical cells. Compile the mod files and run the simulation with its default parameters for each of the four cell types (in the mosinit.hoc file).

   (a) * Modify the hoc code provided on the course website, mainen-sejnowski-1996-problem.hoc, to produce plots of the frequency of firing (based on the last two spikes in the spike train) as a function of injected current. Vary the current in steps from 0 to 150% of the default current used in the model. You should use at least 15-20 steps to get a good picture of the onset of spiking (and you can zoom in with more steps near the current threshold if you wish).

   (b) ** Further modify the code provided to determine the *interburst interval frequency* and plot it as a function of DC injected current amplitude.

5. Singular perturbation theory. Consider the following system of equations, which give a piecewise linear rough approximation of the Fitzhugh-Nagumo equations:

$$\begin{aligned}v' &= f(v) - w && (1)\\ w' &= \varepsilon(v - \gamma w) && (2)\\ f(x) &= \begin{cases} -x-2 & \text{for} \quad x < -1 \\ x & \text{for} \quad -1 \le x \le 1 \\ -x+2 & \text{for} \quad x > 1. \end{cases} && (3)\end{aligned}$$

   (a) ** Implement the equations above in XPP. Start from the fhn.ode file and make whatever changes are necessary. In order to create a piecewise defined function you can use the heaviside step function $\Theta(x)$ which is 0 for $x < 0$ and 1 for $x \ge 0$. The heaviside function in XPP is `heav`. Here's an example that gives the function defined above:

```
f(v)=(-2-v)*heav(-v-1) + v*heav(v+1) + 2*(1-v)*heav(v-1)
```

   Produce plots of the trajectories for $\gamma = 1/2$ and $\varepsilon = 0.2$, 0.05 and 0.01.

   (b) *** Approximate the periodic solution to the system of equations (1-3) above. Estimate the period from your approximate solution and compare with XPP simulations. Use $\varepsilon = (0.15, 0.1, 0.05, 0.02, 0.01)$.

* denotes problems to be done by all students.

** denotes problems to be done students enrolled in MATH 378 or any of the cross-lists numbered 478. Any math or statistics majors and any graduate students enrolled in any section should do these problems as well.

*** denotes problems to be done by students enrolled under MATH 478, or math graduate students enrolled under any course number.