Computational Neuroscience

aka MATH 378/478, BIOL 378/478, EECS 478, NEUR 478, EBME 478, COGS 378

Homework Problems # 4

due Tuesday 4/24/08 in class
ASSIGNMENT OPTIONAL

1. *Consider a model for two coupled phase oscillators θ_1, θ_2 represented by

$$\dot\theta_1 = \omega + \epsilon \sin(\theta_2 - \theta_1) \quad (1)$$
$$\dot\theta_2 = \omega + \epsilon \sin(\theta_1 - \theta_2) \quad (2)$$

for $\epsilon > 0$. Let $\Delta = \theta_2 - \theta_1$ be the phase difference between the two oscillators.

(a) Reduce the equations for $\dot\theta_1$ and $\dot\theta_2$ to a single equation for $\dot\Delta$.

(b) Find all the steady state values of Δ and determine their stability. Describe each steady state qualitatively (synchronous, antisynchronous).

(c) Repeat part (b) under the assumption that $\epsilon < 0$.

2. **Consider a model for three coupled phase oscillators θ_1, θ_2 represented by

$$\dot\theta_1 = \omega + \epsilon \sin(\theta_2 - \theta_1) + \epsilon \sin(\theta_3 - \theta_1) \quad (3)$$
$$\dot\theta_2 = \omega + \epsilon \sin(\theta_3 - \theta_2) + \epsilon \sin(\theta_1 - \theta_2) \quad (4)$$
$$\dot\theta_3 = \omega + \epsilon \sin(\theta_1 - \theta_3) + \epsilon \sin(\theta_2 - \theta_3) \quad (5)$$

for $\epsilon > 0$.

(a) Reduce the equations for $\dot\theta_i$ to a pair of equations for $\dot\Delta_1$ and $\dot\Delta_2$ where $\Delta_1 = \theta_2 - \theta_1$ and $\Delta_2 = \theta_3 - \theta_2$.

(b) Find all the steady state values of $\vec\Delta = (\Delta_1, \Delta_2)$ and determine their stability. (Suggestion: draw a phase plane for Δ_1 and Δ_2.) Describe each steady state qualitatively (synchronous, antisynchronous, etc.)

(c) Repeat part (b) under the assumption that $\epsilon < 0$.

3. ***Consider a model for four coupled phase oscillators with a mixture of "excitatory" coupling ($\eta > 0$) and "inhibitory" coupling ($\epsilon < 0$) represented by

$$\begin{aligned}
\dot{\theta}_1 &= \omega + \eta\,(\sin(\theta_2 - \theta_1) + \sin(\theta_4 - \theta_1)) + \epsilon \sin(\theta_3 - \theta_1) \\
\dot{\theta}_2 &= \omega + \eta\,(\sin(\theta_3 - \theta_2) + \sin(\theta_1 - \theta_2)) + \epsilon \sin(\theta_4 - \theta_2) \\
\dot{\theta}_3 &= \omega + \eta\,(\sin(\theta_4 - \theta_3) + \sin(\theta_2 - \theta_3)) + \epsilon \sin(\theta_1 - \theta_3) \\
\dot{\theta}_4 &= \omega + \eta\,(\sin(\theta_1 - \theta_4) + \sin(\theta_3 - \theta_4)) + \epsilon \sin(\theta_2 - \theta_4)
\end{aligned}$$

(a) Draw a diagram depicting the four phase oscillators and their coupling arrangement. What is the symmetry group of the system?

(b) Reduce the equations for $\dot{\theta}_i$ to a set of three equations for $\dot{\Delta}_i, i = 1, 2, 3$ where $\Delta_i = \theta_{i+1} - \theta_i$.

(c) Find all the steady state values of $\vec{\Delta} = (\Delta_1, \Delta_2, \Delta_3)$ and determine their stability. Describe each steady state qualitatively (synchronous, antisynchronous, etc.)

(d) Repeat part (b) under the assumption that $\epsilon < 0$.

4. * The Sheffer Stroke function, written $A|B$, is a "functionally complete" logical operator in the sense that the usual binary operators (not, and, or, implies) can be expressed in terms of it. Logical implication ("if A then B") is more familiar, and is written $A \to B$. Design a McCulloch-Pitts network that calculates the Sheffer Stroke function of two inputs, A and B, and another that calculates logical implication. These functions have the following truth tables:

$(A\|B)$	A=0	A=1
B=0	1	1
B=1	1	0

$(A \to B)$	A=0	A=1
B=0	1	0
B=1	1	1

* denotes problems to be done by all students.

** denotes problems to be done students enrolled in MATH 378 or any of the cross-lists numbered 478. Any math or statistics majors and any graduate students enrolled in any section should do these problems as well.

*** denotes problems to be done by students enrolled under MATH 478, or math graduate students enrolled under any course number.