
Birkhoff, Garrett
Chevalley, Claude
Dieudonné, Jean
Mac Lane, Saunders (co-authored with William Lawvere)
Weil, André
Fifty Years


Dedicated to students and colleagues Caroline H. MacGillavry, Jan A. Ketelaar, Eelko H. Wiebenga, and Aafje Looijenga-Vos.


Birkhoff was the son of mathematician George David Birkhoff and Margaret Grafius Birkhoff. George Birkhoff, the father, was the first American mathematician to gain wide respect in Europe. Garrett Birkhoff is more remembered for promoting new conceptions than specific theorems. His most important single result was a theorem that instituted a conception, the Birkhoff variety theorem, originating modern universal algebra. He showed the power of deceptively simple algebraic properties and the feasibility of more complex and realistic applied mathematics, and he was among the first mathematicians to rely heavily on computers.

Lattices and Universal Algebra. Entering Harvard College in 1928, Birkhoff aimed at mathematical physics. Physics led him to partial differential equations, which in turn led to more abstract ideas, including Lebesgue theory and point-set topology. Curiosity led him to finite groups. After graduating in 1932, he went to Cambridge University for physics. That July, though, he visited Munich and met Constantin Carathéodory, who pointed him towards algebra and especially van der Waerden's great new textbook Moderne Algebra (Berlin: Springer, 1930). Back in Cambridge he switched to algebra with group theorist Philip Hall.

Birkhoff turned the study of subgroups, subrings, and so on into two branches of mathematics. The intersection $H \cap K$ of subgroups of a single group $G$ is also a subgroup of $G$. The union $H(K)$ of subgroups of $G$ is generally not a subgroup because an element of $H$ and another of $K$ may...
combine to give one that is in neither. Yet $H$ and $K$ will generate a subgroup $HK$ which is called their join, defined as the smallest subgroup of $G$ that contains both $H$ and $K$, and so is generally larger than the set theoretic union. This suggests a dual definition: the meet $H \cap K$ is the largest subgroup of $G$ contained in both $H$ and $K$. In fact, the meet of subgroups is their intersection, but other structures than groups can have meets that are smaller than intersections. In England Birkhoff organized and generalized the study of such order relations into lattice theory. He also characterized a wide array of structures whose substructures form lattices and organized their study as universal algebra. Each subject had precedents, notably in the work of Richard Dedekind and Emmy Noether, but Birkhoff established them as subjects.

Birkhoff enjoyed the unity and economy of the abstract idea of an order relation on a set. That is any relation $xy$ on the elements of the set such that: 1) $xby$ for all elements $x$ of the set; 2) for any elements $x,y,z$, if $xby$ and $ybz$ then $xbyz$; and 3) for any elements $x,y$, if $xby$ and $yb$ then $x = y$. The relation $xy$ is usually read "$x$ is less than or equal to $y"$ although it may have nothing to do with magnitude. He gave the example of logical propositions with $xy$ defined to mean $x$ implies $y$, and $x = y$ defined to mean that $x$ is logically equivalent to $y$. He defined a lattice as an ordered set where every two elements $x,y$ have a join $x\wedge y$ defined as the smallest element greater than or equal to both $x$ and $y$, and a meet $x\vee y$ defined as the largest element less than or equal to both $x$ and $y$, with a few further properties. Logical propositions form a lattice where the join $x\wedge y$ of propositions is their disjunction "$x$ or $y"$ and their meet $x\vee y$ is the conjunction "$x$ and $y"$. The subgroups of a group $G$ form a lattice when $HKB$ is defined to mean $H$ is contained in $K$. The notion of lattice is wide enough to include many examples yet specific enough to yield many theorems. Birkhoff also found further abstract conditions characterizing various kinds of lattice. His 1940 book Lattice Theory is still in print, with new concepts and results tripling its original length.

Not all mathematical structures are as tidy as groups. The substructures of a given structure do not always form a lattice. So, which ones do? Birkhoff found an elegant sufficient condition.

A Birkhoff variety is a class containing all the structures defined by a given set of operators and equations. For example, a commutative ring is a set $R$ with a selected zero element 0 and unit element 1 and addition, subtraction, and multiplication $+,-,\cdot$; satisfying equations familiar from arithmetic, such as the zero law $x+0=x$ and the commutative law for addition $x+y=y+x$. These equations are understood to hold for all elements $x,y$ of $R$. A field is a commutative ring $R$ meeting a further more complex condition, not an unrestricted equation but a conditional equation: If $x \neq 0$ then $x$ has an inverse, an element $y$ in $R$ with $xy = 1$. A variety is a class of structures definable purely by operators and equations, as shown above for the class of commutative rings and not for the class of fields.

Varieties enjoy very special properties compared to other classes of structures. For example, any two rings $R,S$ have a product $R \times S$. An element of $R \times S$ is an ordered pair $x,u$ with $x$ and element of $R$ and $u$ and element of $T$. The zero element of $R \times S$ is the pair of zeros 0,0, the unit is the pair of units 1,1. The operations are defined component-wise, which for addition means $x \cdot u + y \cdot v = x+y \cdot u+v$.

The analogues hold for subtraction and multiplication, and $R \times S$ satisfies all the ring equations since it satisfies them all in each component. The same does not work for fields. Even if $R$ and $S$ are both fields, $R \times S$ is not. Its element 1,0 is not zero because the first component is not the zero of $R$ but it has no inverse either because the second component has no inverse in $S$.

The Birkhoff variety theorem lists a few constructions such as products, and proves that a class of structures is a variety if, and only if, it is closed under these operations. Given any class of structures, no matter how it was originally defined: it can be defined purely by equations if, and only if, these listed constructions apply to it and always yield results in that class. Because fields do not have products there cannot be any way to characterize fields purely by equations. These constructions imply that the substructures of any structure in a variety form a lattice. The theorem created modern universal algebra defined as the study of Birkhoff varieties. Earlier more sweeping universal theories of algebra were not so productive as Birkhoff’s.

**A Career at Harvard.** In 1933, Birkhoff returned to Harvard as a member of the Society of Fellows, and in 1936 he joined the mathematics department. He never earned a doctorate. He married Ruth Collins in 1938 and eventually had two daughters and a son (Ruth, John, and Nancy). He began teaching the new abstract algebra, which Saunders Mac Lane also taught there. Their 1941 *Survey of Modern Algebra* was the first effective English language introduction to the material of van der Waerden’s *Moderne Algebra* and was an immediate success. It was augmented by the 1967 *Algebra* with the order of the authors’ names reversed and more emphasis on category theory. These two books had a huge impact on mathematics students for fifty years and continued to shape the standard U.S. algebra curriculum in the early twenty-first century.

During World War II, Birkhoff worked on fluid dynamics, including the explosion of bazooka charges and
problems of air-launched missiles entering water. Chapters of his 1950 book Hydrodynamics: A Study in Logic, Fact, and Similitude were named for various "paradoxes" where either the models idealize phenomena in unrealistic ways or basically plausible models give some bizarre results. Birkhoff emphasized group theory for handling symmetries in hydrodynamics, although one paradox was the breakdown of symmetries in some realistic hydrodynamic situations. He urged innovative numerical methods and his later more specialized hydrodynamics relied more heavily on computing.

He consulted on reactor design for the Bettis Atomic Power Laboratory from 1955 to 1961, working on numerical solutions for partial differential equations by repeatedly improving successive approximations. Starting in 1959 he consulted for General Motors on numerical description of surfaces, to guide numerically controlled machinery cutting the dies used to stamp out automobile body parts. This led him to major contributions to "spline" methods fitting segments of cubic polynomials to data points.

Birkhoff was elected to the American Academy of Arts & Sciences in 1945, the American Philosophical Society in 1960, and the National Academy of Sciences in 1968. He received honorary degrees from the National University of Mexico, the University of Lille, and Case Institute of Technology.

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Describes the history and influence of Birkhoff's lattice theory and universal algebra in several places; see the index.


Colin McLarty

BJERKNES, VILHELM (b. Christiania, Norway, 14 March 1862; d. Oslo, Norway, 9 April 1951), meteorology. For the original article on Bjerkes see DSB, vol. 2.

Frequently called the father of modern meteorology, Bjerknes reluctantly devoted himself to atmospheric science. His scientific career, beginning in the 1890s, reveals an astute scientist willing to overcome professional marginalization by developing skills as a disciplinary entrepreneur. Beginning as a theoretical physicist devoted to a mechanical worldview, he hesitantly turned to creating a physics of the atmosphere and oceans. In 1897 he elaborated a hydrodynamic equation for circulation in fluids in which density could depend upon several variables. He soon understood that motions in the atmosphere and oceans could be comprehended through this theorem: He ultimately developed a physical hydrodynamics that became a basis for dynamic meteorology and oceanography. The capstone of his career, the creation of the so-called Bergen School of meteorology, established a new conceptual foundation for the science while creating innovative predictive practices that enabled greater integration of weather as a resource for agriculture, aviation, and fishery.

Career Strategies. Bjerkes's professional options in physics were limited by both his disposition and his circumstances. Bjerkes hoped to achieve a mechanical depiction of the ether, which would vindicate his father Carl Anton's hydrodynamic analogies to electromagnetism and serve as an illustration of the contiguous-action physics proposed by Heinrich Hertz. He thought this research would remain central to European physics and could be his vehicle to prestige and authority, but he was mistaken. He felt helpless in the early 1900s when, to his mind, German-speaking theoretical physicists in a state of mass psychosis abandoned the mechanical worldview for electromagnetic alternatives. In Sweden there was little sympathy for theoretical physics, and as a Norwegian working in Stockholm at a time of tensions between the two nations, he found his options limited.

The most resolute modernizer among the founders of Bourbaki, and the most given to austere axiomatic abstraction, Chevalley was influential in setting the broad agenda of Bourbaki’s project and for major advances in number theory and the theory of Lie groups. He was the chief representative of mathematical logic in France in the 1930s not for any work of his own but for promoting the ideas of his friend Jacques Herbrand, who died in a mountain-climbing accident in 1931 at age twenty-three. Chevalley had philosophical, cultural, and political interests, in which “his mathematician friends had the impression that he proceeded as in mathematics, by the axiomatic method: having posed some axioms he deduced consequences from them by inflexible logic and unconcerned with obstacles along the route which would have led anyone else to go back and change the axioms” (Dicudonné, 1999, p. 113).

Early Life and Education. Chevalley was born into socially rising French Protestant circles little more than a hundred years after Protestants gained full citizenship. His paternal grandfather was a Swiss-born clockmaker naturalized as French. His uncles on that side were a legal councilor to the king of Egypt and a head doctor at l’Hôpital des Enfants-Malades in Paris (France’s first children’s hospital). His father, Daniel Abel Chevalley, passed to become professor of English at the Lycées Voltaire and Louis-le-Grand in Paris. He became a diplomat in South Africa, Norway, and the Caucasus and Crimea, and returned to academic research after retirement. Chevalley’s maternal grandfather was a village pastor in the Ardèche who became a professor of English at the Lycées Voltaire and Louis-le-Grand in Paris. He was an Anglicist and coauthor with her husband of the first edition of the Concise Oxford French Dictionary. His parents married in 1899 and besides their son had a daughter Lise, who lived to have children but died in 1933. Daniel also died in 1933, while Marguerite lived to 1969. They were active in the Association France-Grande-Bretagne—a group founded during World War I and prominent enough that when the Germans took Paris in 1940 there was barely time to destroy the records before the Gestapo came for them.

Chevalley traveled with his parents until he began school at Chançay and then in Paris at the Lycée Louis-le-Grand, where he gained his love of mathematics and

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began to study the standard analysis textbook, Édouard Goursat’s 1902 *Cours d’analyse*. In 1926 he entered the École Normale Supérieure (ENS) in Paris at the early age of seventeen. He formed an important friendship with Herbrand, who had also been admitted at seventeen, one year before him. The two were drawn to number theory, and to German mathematics, in 1927 when they met André Weil. They took courses from the same excellent but dated mathematicians as Weil. Jacques-Salomon Hadamard’s seminar gave them glimpses of recent mathematics, and they taught themselves more from original sources than they learned in courses. Chevalley studied especially with Émile Picard, graduated in 1929, and published a note on number theory in the *Comptes Rendus de l’Academie des Sciences* that year. In 1929–1930 Herbrand did his military service and produced the work that Chevalley later said formed the basis of the new methods in class field theory. Herbrand spent 1930–1931 in Germany studying logic, and died on holiday in the Swiss Alps on the way back. Chevalley spent 1931–1932 studying number theory especially with Emil Artin at Hamburg and Helmut Hasse at Marburg and came back to earn his doctorate from the University of Paris in 1933 with a thesis on the work he did in Germany.

**Class Field Theory.** Class field theory had been at the top of the agenda in number theory since Teiji Takagi around 1920 proved a series of decades-old conjectures of Leopold Kronecker and David Hilbert. The proofs were extremely complicated, and the results were at once productive of concrete arithmetic theorems and promising of further theoretical advances. The subject was prestigious and daunting. Chevalley’s work on it showed a typical difference from Weil, who used modern number theory to expand themes from classical analysis while Chevalley worked to remove analysis in favor of algebra and point-set topology. Weil’s impulse was more classically geometrical and Chevalley’s more algebraic.

Carl Friedrich Gauss already used what are now called the Gaussian numbers $\mathbb{Q}[i]$ in arithmetic. These are expressions $p+qi$ where $p$ and $q$ are any ordinary rational numbers and $i$ is the imaginary square root of $-1$. In particular the Gaussian integers $\mathbb{Z}[i]$ are those $a+bi$ where $a$ and $b$ are any ordinary integers. An ordinary prime number may not be prime in the Gaussian integers, as for example the ordinary prime 5 factors as $5=(2+i)(2-i)$. Yet the Gaussian integers do have unique prime factorization analogous to the ordinary integers. This is the prime factorization of 5 as a Gaussian integer. Using other algebraic irrationals in place of $i$ gives other algebraic number fields $\mathbb{K}$ in place of the Gaussian numbers $\mathbb{Q}[i]$. And each algebraic number field $\mathbb{K}$ contains a ring $\mathbb{A}$ of algebraic integers analogous to the Gaussian integers $\mathbb{Z}[i]$ although generally not so easy to describe. If rings of algebraic integers always had unique prime factorization, then there would be wonderful consequences, such as a one-page proof of Fermat’s last theorem. They do not. Class field theory began as an astonishing way to measure and work with failures of prime factorization.

Each algebraic number field $\mathbb{K}$ extends to a certain larger field $L(\mathbb{K})$ called the Hilbert class field, so that the Galois group of $L(\mathbb{K})$ over $\mathbb{K}$ measures the failure of unique prime factorization in the ring $\mathbb{A}$. Furthermore, roughly speaking, all the algebraic integers in $\mathbb{A}$ have unique prime factorization in $L(\mathbb{K})$. If $\mathbb{A}$ itself has unique prime factorization then $K=L(\mathbb{K})$ and the Galois group is the trivial $[1]$. A nontrivial Galois group for $L(\mathbb{K})$ shows failure of prime factorization in $\mathbb{A}$, and a larger Galois group shows greater failure. Explicit descriptions of Hilbert class fields were known—some using classical complex analysis. Other arithmetic properties of the ring $\mathbb{A}$ are expressed by the Galois groups of other extensions of $\mathbb{K}$, which are also called class fields of various kinds.

The German number theorists would study any given algebraic number field $\mathbb{K}$ and its ring of algebraic integers $\mathbb{A}$ in connection with other related fields called local fields, so-called because these fields often concentrate attention on a single prime factor. One of Chevalley’s typical contributions was to stress the sense in which every one of them concentrates on a single prime—if, for example, the rational number field $\mathbb{Q}$ is taken to include one “infinite prime” along with the finite primes 2, 3, 5, 7, and so on. A less vivid term for infinite primes is Archimedean places. For each ordinary prime number $p$ the $p$-adic numbers $\mathbb{Q}_p$ focus on the single factor $p$. From this point of view the real numbers $\mathbb{R}$ focus on absolute value, which at first glance is nothing like a prime factor, but there are extensive axiomatic analogies. Chevalley stressed how much simpler the theory becomes when “infinite primes” are put on a par with the finite.

The theory of prime factorization in any one local field is extremely simple because there is only one prime. The algebraic number fields $\mathbb{K}$ are global, as each one of these fields involves all the usual finite primes at once plus some infinite. Number theorists would calculate various class fields for $\mathbb{K}$ by quite complicated use of related local fields. Chevalley multiplied the number of basic definitions manifold and yet simplified, unified, and extended the theory overall by introducing class fields directly for the local fields themselves. He earned his doctorate from the University of Paris in 1933 with a thesis on class field theory over finite fields and local fields.

The dissertation won Chevalley research support in Paris through 1937. For several reasons he worked on eliminating classical complex analysis from class field theory: the local fields related to prime numbers $p$ favored
algebraic methods. Chevalley extended ideas from Wolfgang Krull and Herbrand to unify class field theory by generalizing algebraic number fields $K$ to infinite degree extensions of $\mathbb{Q}$ (that is, extensions involving infinitely many independent algebraic irrationals) and thus also favored algebraic tools. And Chevalley was personally drawn to modern, pure, uniform algebraic methods replacing classical complex analysis.

Chevalley advanced each of these goals by his creation of \textit{id\textbar les} linking local and global. Roughly speaking, an \textit{id\textbar les} of a global field $K$ is a list of ways that a nonzero element of $K$ might look in each of the local fields related to $K$. This list may or may not actually be generated by an element of $K$, just as an ideal of a ring may or may not be generated by a single ring element. So \textit{id\textbar les} are more flexible and more easily accessible than elements, and arithmetic conclusions about $K$ follow from knowing which \textit{id\textbar les} correspond to elements. Weil added a related notion of \textit{ad\textbar les}, and the two notions are today central to algebraic number theory.

Local fields and the Galois groups of infinite degree field extensions both have pro-finite topologies. For example, in the $p$-adic integers $\mathbb{Z}_p$, a number counts as closer to 0 when it is divisible by a higher power of $p$ (that is, when it equals 0 modulo a higher power). This was a triumph for the axiomatic definition of a topological space. All of these spaces satisfy the axioms. Topological notions of continuity, compactness, and so forth, are very useful in studying them. Yet they are far from ordinary spatial intuitions. For one thing, they are everywhere disconnected. The only connected parts of such a space are the single points, and yet these disconnected points "cluster around" one another. Chevalley applied the axioms completely unconcerned with classical geometric intuitions. \textit{id\textbar les} also have an algebraically defined topology, although a version by Weil suited to Fourier analysis has replaced Chevalley's version.

\textbf{Bourbaki.} Hadamard's seminar ended in 1933. Gaston Julia allowed Weil, Chevalley, and other ENS graduates to run a new seminar in his name on recent mathematics ignored by the dominant mathematicians of Paris at the time. These meetings gave rise to a plan to replace the venerable Goursat (1902) by a new up-to-date textbook on analysis. And so Henri Cartan, Claude Chevalley, Jean Delsarte, Jean Dieudonné, René de Possel, and André Weil met on Monday 10 December 1934 at the Café Capoulade. The textbook project quickly expanded to a project for a book series on the basics of all pure mathematics, which would reestablish French dominance in mathematics. The group adopted the collective pseudonym of Nicolas Bourbaki, and the series became the phenomenally influential \textit{Elements of Mathematics}. Chevalley's mother hosted two early Bourbaki congresses at the family property at Chançay.

Dieudonné says that Chevalley was early a leader because "at the start only Weil and he had the vast mathematical culture required to conceive a plan for the whole, while each of the others only gradually acquired the necessary panoramic view" (1999, p. 107). Probably through Herbrand's influence, Chevalley was an ardent advocate of Hilbert-school axiomatic rigor indifferent to any non-mathematical reality. This was common ground in Bourbaki, and Bourbaki had a large role in making it widespread today—despite complaints in some quarters that Bourbaki's \textit{Elements} promoted sterile abstraction. Chevalley outdid even Weil in this style, so that Weil reviewed Chevalley as "algebra with a vengeance; algebraic austerity could go no further ... a valuable and useful book ... [yet] severely dehumanized" (1951).

Chevalley did not see himself that way. He joined the personalist group "l'Ordre Nouveau," not to be confused with a later extreme right group by the same name. They promoted personal liberty and growth and rejected all of anarchism, despotism, Marxism, and "le capitalisme sauvage," or unrestrained capitalism.

\textbf{Career.} The single result most widely associated with Chevalley is the Chevalley-Warning theorem, far from his deepest theorem, and suggested by a less specific conjecture of Artin. The name credits Chevalley for seeing its importance as early as 1936. Any finite field has some prime number $p$ as characteristic and then the number of elements is some power $p^n$ of $p$. Chevalley's part of the theorem says: for any polynomial $P(X_1, \ldots, X_t)$ with degree less than the number of variables, the number of roots in any finite field $\mathbb{F}$ is divisible by the characteristic of $\mathbb{F}$. In particular, every homogeneous polynomial has a solution $\langle 0, \ldots, 0 \rangle$ so if the degree is less than the number of variables then it has nonzero solutions. This is important to projective geometry over finite fields.

Chevalley did his first teaching in 1936 at Strasbourg, replacing Weil, who had gone to the Institute for Advanced Study in Princeton, New Jersey, and then at Rennes in a teaching-research position (\textit{maître de conférences}) 1937–1938, replacing Dieudonné who had gone to Nancy. In 1938 he was invited to the Institute for Advanced Study and he was there when World War II broke out in Europe. The French ambassador felt he might best serve France by remaining in the United States, where he was the only French scholar at the time.

Princeton University made him a professor, and he remained there until 1948, when he moved to Columbia University, where he stayed until 1955. American students found him terse and demanding, so that few undertook research with him. In 1933 Chevalley had married his
first-cousin Jacqueline. There were no children, and the marriage dissolved in 1948. That same year he married his second wife, Sylvie, a professor and theater historian in New York. Their daughter Catherine was born in 1951 and became a philosopher and historian of science. When the family returned to France, Sylvie would become librarian and archivist for the Comédie Française.

In the 1940s Chevalley took up algebraic geometry and Lie groups and most especially the union of these themes in the Lie algebras of algebraic groups. He followed Weil in seeking algebraic geometry not only over the complex numbers but over an arbitrary field. In other words he wanted to eliminate classical complex analysis here too, in favor of purely algebraic methods. He would return to this in his 1950s Paris seminar, described below.

His greatest mathematical influence was in Lie groups and centered on his use of global topological methods regarding a Lie group as a manifold. Lie groups had always been defined as (real or complex) manifolds that are simultaneously groups, but workers from Sophus Lie (1842–1899) on had routine methods for looking at the group operation in an infinitesimal neighborhood of the unit element, while they looked at the space as a whole on a more ad hoc basis as needed. In precise terms, they had better tools for Lie algebras than for Lie groups directly.

The real line $\mathbb{R}$ is a Lie group with real-number addition as group law, and the unit circle $S^1$ is a Lie group with angle addition as group law. A small neighborhood of the unit is the same in the two cases: In $\mathbb{R}$ it is 0 plus or minus some small real number. In $S^1$ it is 0° plus or minus some small number of degrees. So $\mathbb{R}$ and $S^1$ have the same Lie algebra. The difference is global: traveling away from 0 in $\mathbb{R}$ leads to ever new real numbers; while traveling away from 0° in $S^1$ eventually circles back to 0°. Lie thought of his groups globally. Henri Poincaré (1854–1912) created his tools for topology largely to handle the global topology of Lie groups. Chevalley's Princeton colleague Hermann Weyl made group representations central to his work, reflecting the global properties of all the groups that cannot be reduced in a certain way to simple groups. But no one before Chevalley succeeded in making global topology so explicitly fundamental to Lie group theory.

On the one hand this organized the theory so well that Chevalley (1946) became "the basic reference on Lie groups for at least two decades" (Dieudonné and Tits, 1987, p. 3). On the other hand it combined with Chevalley’s interest in finite fields and algebraic geometry. As the global theory was less analytic, it could generalize. An algebraic group is rather like a Lie group, possibly over some field other than the real or complex numbers. Each algebraic group has a kind of Lie algebra, but over most fields not all of these Lie algebras come from groups. Chevalley found a remarkable procedure whereby every simple Lie algebra (that is, one that is not abelian and contains no nontrivial ideals) over the complex numbers corresponds to a simple algebraic group over any field (that is, a group with no nontrivial quotients). A huge feat in itself, this fed into one of the largest projects in twentieth-century mathematics. Applied to any finite field it gives finite simple groups now called Chevalley groups; it was a useful organizing device, and gave some previously unknown finite simple groups. It became a tool in the vast and now completed project of describing all the finite simple groups. Further Chevalley hoped his algebraic group methods applied to algebraic number fields and related fields would be useful in arithmetic. Alexander Grothendieck’s algebraic geometry soon made them so.

A Guggenheim grant took Chevalley back to Paris for the year 1948–1949. He happily rejoined the Bourbaki circle and shared research with them. He was also happy to leave the anticommunist atmosphere of America during the 1948 Soviet blockade of Berlin, the 1949 Soviet atomic bomb test, and the imminent rise of McCarthyism. A Fulbright grant took him to Japan for 1953–1954, including three months at Nagoya. Chevalley had worked with Japanese mathematicians in Hamburg and later in Princeton and always maintained close ties to Japan.

**Return to France.** A 1954 campaign to place Chevalley at the Sorbonne "unleashed passions in the mathematical community scarcely comprehensible today" (Dieudonné, 1999, p. 110). Some believed new opportunities should go to those who had stayed and fought or been captured in the army or the resistance. Others opposed Bourbaki’s influence. Chevalley was appointed in 1955 to the nearby Université de Paris VII, where he retired in 1978.

He began a seminar in 1956 at the École Normale Supérieure, initially with Cartan, and made it an early home for Grothendieck’s project to rewrite the foundations of algebraic geometry. Chevalley among others anticipated aspects of Grothendieck’s scheme theory, and the seminar proceedings include the first published use of the word *scheme* (*schéma*) in its current sense in algebraic geometry. He claimed “Grothendieck had advanced algebraic geometry by fifty years” (Seshadri, 1999, p. 120).

Chevalley continued to deepen his work on groups. He determined all of the semisimple groups over any algebraically closed field, that is, all of the groups that cannot be reduced in a certain way to simple groups. When the field has characteristic 0, or in other words contains a copy of the rational numbers $\mathbb{Q}$, both the result and the proof were very like those known fifty years before over the complex numbers using methods of Lie algebras. But when the field has a finite prime characteristic $p$, so that it contains a copy of the finite field of $p$ elements, the older proof did not work at all. Lie algebras in finite characteristic were not fully classified themselves, they were known...
to be much more complicated than in characteristic 0, and Chevalley found they tell much less about the Lie groups. Surprisingly, then, when Chevalley completed the proof for finite characteristic the result was independent of which finite characteristic, and rather parallel to characteristic 0. Since Lie algebra methods were unusable, he produced purely group theoretic and algebra-geometric methods and the deepest work of his career. In the 1960s he turned to more detailed work on finite groups and produced and inspired new developments on them.

He retained his philosophical and political ideas though he had lost the friends who shared them; the last of his philosophical friends, Albert Lautman, died in the Resistance. Having become skilled at the game of Go in Japan, Chevalley promoted it among his friends and students, some of whom organized the first Go club in France in 1969. He supported the student movement of 1968 and charitable progressive projects of a kind associated with French Protestantism though he did not keep the religious faith. Chevalley had small respect for academic honors. He accepted the Cole Prize of the American Mathematical Society in 1941, and honorary membership in the London Mathematical Society in 1967. Dieudonné wrote, "No one who has known Chevalley can fail to see how his [philosophical-political] principles agreed with his entire character. When his articles exalt ‘revulsion at accommodations, at self-satisfaction or satisfaction with humanity in general, and at every kind of hypocrisy’ he describes himself" (1999, p. 112).

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**Works by Chevalley**


**Chevalley**


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*Colin McLarty*

### Chioniades, George (or Gregory) (b. Constantinople, c. 1240–1250; d. Trebizond, c. 1320), astronomy.

Byzantine scholar, physician (iatrosophist), and astronomer, George Chioniades was born probably at Constantinople between 1240 and 1250 and died at Trebizond around 1320. His life is not well known. According to his correspondence, it is known that he traveled between Constantinople, Trebizond, and Tabriz during 1295 to 1310. Monk and priest, he was named bishop of Tabriz, probably around 1304 or 1305, to defend Christians living in the Mongol Empire. The change of name from George to Gregory may have occurred at that time. It may have been on the occasion of this nomination that he wrote a profession of Christian faith, preserved in Vaticanus gr. 2226. In this document, he refuted accusations of having adopted foreign beliefs from having stayed so long among the Persians, the Chaldeans, and Arabs. He also defended himself against having accepted astrological fatalism contrary to the Christian religion. Around 1310 or 1314, he spoke of himself as an old man. On his death some of his classical books passed into the hands of...


**OTHER SOURCES**


**Joanne Bourgeois**

**DIEUDONNÉ, JEAN** (b. Lille, France, 1 July 1906; d. Paris, France, 29 November 1992), analysis, algebra, history of mathematics, Bourbaki.

Dieudonné was distinguished as much for vast mathematical knowledge as for his own innovations. He influenced twentieth-century mathematics through his role in the Bourbaki group, his nine-volume *Treatise on Analysis*, his four-volume collaboration with Alexander Grothendieck, and his historical writing. He wrote tens of thousands of pages. More than any other mathematician in Bourbaki, Dieudonné stressed the simplifying role of axiomatics, which David Hilbert also stressed, as opposed to the generalizing role. Fundamental, classical theorems often use far less than the classical assumptions. Dieudonné would drop the irrelevant assumptions and prove the theorems from just the relevant axioms. Of course the results are also more general, but Dieudonné aimed more to organize and unify than to generalize.

**Youth.** Dieudonné deeply admired and respected his father, Ernest, who had supported a family from the age of twelve and rose from a modest employee to become general director of a textile manufacturing group. Ernest valued education, strove to make up for what he had missed, and married Léontine Lebrun who taught grade school until Jean was born. A few years later came a sister, Anne Marie. Jean's mother taught him to read before he went to school. He favored dictionaries, encyclopedias, and universal histories. He began school in Lille, but in 1914 when the city declared itself indefensible and surrendered to the Germans he went to Paris to the Lycée Condorcet.

He spent 1919–1920 as a fellow of Bembridge School on the Isle of Wight to learn English. The school was founded in 1910 on the principles of John Ruskin, who would address social ills by "making a carpenter ... happier as a carpenter" and making the elite a better elite (Hicks, 1974, p. 57). Dieudonné encountered algebra there and found his calling as a mathematician.

With the war over, he returned to Lille and the Lycée Faidherbe. In 1924 he was accepted at both the École Polytechnique and the École Normale Supérieure (ENS) and chose the latter. There he took courses from great mathematicians at the Sorbonne and the Collège de France, notably including C. Emile Picard and Jacques Hadamard. These were older men; France had lost the intermediate generation in World War I. Hadamard's seminar raised topics from recent mathematics, but most instruction was in nineteenth-century analysis. Dieudonné graduated and took first place in the mathematics agrégation examination for teaching at a lycée. Accepted to the doctoral program at the ENS, Dieudonné did his military service from 1927 to 1928 and began research.


In 1931 Dieudonné completed his thesis at the ENS supervised by Paul Montel. He calculated bounds on the locations of zeros of a complex meromorphic function $f(z)$ or its derivative $f'(z)$ given specified bounds for values of $f(z)$ on specified domains. This was the subject of virtually all his publications prior to Bourbaki.

With his record at the ENS, Dieudonné was extremely employable even in hard times. His role in
Bourbaki made him a sought-after professor, although he had no interest in and small gift for teaching. He would eventually hold professorships at five universities in the United States and France, plus visiting professorships at eleven universities in Europe, Asia, and North and South America. He began as an instructor at Bordeaux in 1932 and went to Rennes from 1933 to 1937, first as an instructor and then in a teaching-research position (maître de conférences).

Bourbaki. In the fall of 1934, Odette Clavel dropped her program at a Sunday afternoon concert. Dieudonné picked it up, handed it to her, and married her on 22 July 1935. Fifty-six years later he described the marriage as fifty-six years of happiness. They had a son and a daughter: Jean-Pierre and Françoise. Dieudonné admitted to taking too much time away from them for work.

On Monday, 10 December 1934, Dieudonné joined a handful of ENS graduates called by André Weil to the now-vanished Café Capoulade to plan a thoroughly collaborative, definitive new analysis textbook. It would not have separate chapters by separate experts, but the whole group would write every part of it using the latest tools and the latest standards of rigor out of Hilbert’s school in Germany. It would reestablish French preeminence in mathematics. The others at the café were Henri Cartan, Claude Chevalley, Jean Delsarte, and René de Possel.

The mutation of their project foreshadowed their eventual impact on mathematics. They aimed to replace the text they had all studied and taught, Edouard Goursat’s 1902–1905 Cours d’Analyse Mathématique. Goursat reached classical problems, notably envelopes of families of curves or families of surfaces, the Dirichlet problem, the heat equation, and Fredholm’s equation, by sketchy methods of real and complex algebra, real and complex integration, and formal power series. The young mathematicians quickly saw that rigor would mean vast prerequisites. The projected analysis text turned into an entirely self-contained multivolume work on the methods most widely used across mathematics.

They formed a society under the fictitious name of Nicolas Bourbaki and began the Elements of Mathematics with volumes on set theory, algebra, topology, functions of one real variable, topological vector spaces, and integration. Dieudonné always insisted that the Elements are not encyclopedic because they select only the most useful generalities and reach no serious theorems, but they became the encyclopedia of a new conception of mathematics organized around methods rather than theorems. Despite many critics then and now, the new organization of mathematics became the worldwide norm for graduate training as it proved to be more accessible to students and finally more productive of new great theorems.

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The methodical axiomatic style spread so quickly to other authors that a story has grown that the Elements themselves never worked as textbooks. But there were circles in the 1950s where, as Pierre Cartier recalled, “every time that Bourbaki published a new book, I would just buy it or borrow it from the library, and learn it. For me, for people in my generation, it was a textbook. But the misunderstanding was that it should be a textbook for everybody” (Senechal, 1998, p. 25).

Dieudonné personified Bourbaki. He was a powerful personal force within the group: He worked as a kind of sergeant at arms and did much of the writing. Sections of the Elements went through repeated drafts by different members and were critiqued by all, but Dieudonné wrote every final draft as long as he was an active member. The only works under the name of Bourbaki not approved by the group were the conceptual papers by André Weil (Bourbaki, 1949) and by Dieudonné (Bourbaki, 1950), and Weil and Dieudonné’s historical notes to the Elements (Bourbaki, 1960). Pierre Cartier says: “When Dieudonné was the ’scribe of Bourbaki’ every printed word came from his pen. With his fantastic memory he knew every single word. You could say ‘Dieudonné what is the result about so and so?’ and he would go to the shelf and take down the book and open it to the right page. After Dieudonné retired no one was able to do this” (Senechal, 2005, p. 28).

Dieudonné often praised the way collaboration reshaped his research: “if I had not been submitted to this obligation to draft questions I did not know a thing about, and to manage to pull through, I should never have done a quarter or even a tenth of the mathematics I have done” (1970, p. 144). He published a little more on zeros of functions, but Bourbaki took him into abstract algebra and point-set topology and a modicum of the new logic.

With Henri Cartan, Dieudonné wrote a series of notes on teratopology or counterexamples to plausible guesses in point-set topology. He began work that he would later extend with Laurent Schwartz on topologies for infinite dimensional vector spaces with applications to functional analysis.

Dieudonné’s innovations were often extremely useful without being deep or hard. He gave the idea of paracompact topological spaces, where every open cover has some open locally finite refinement—that is, a cover by open subsets of sets in the original cover and such that any point lies in just finitely many of these subsets; he proved every separable metrizable space is paracompact. He defined partitions of unity for covers. That is, on suitable spaces (which, depending on details, are basically the paracompact spaces) given any locally finite cover of the space by open subsets $U_i$ each set $U_i$ of the cover can be assigned a smooth function $f_i$ which is 0 outside $U_i$ and bounded between 0 and 1 inside it, and at every point the
sum of the values of the functions is 1. A partition of
unity on a cover gives a systematic way to take local con-
structions on each set of the cover and add them together
to get smooth constructions on the whole space.

Bourbaki is especially identified with the idea of a
mathematical structure. Dieudonné was clear: "I do not
say it was an original idea of Bourbaki—but there is no
question of Bourbaki's containing anything original" (1970, p.
138). But Dieudonné and Weil led the group in codifying
ways that a few kinds of structure recur throughout math-
ematics; for example, the addition of real numbers, and of
vectors, and multiplication of matrices are all associative
binary operations. Or, for another example, divisibility of
integers and inclusion of subsets are both transitive rela-
tions. Bourbaki from the first volume of the Elements in
1939 sought a general theory of all the ways a set can be
structured: by operations on the set, or relations among
its members, or a topology on the set. But the theory they
produced was not general enough to apply to all the math-
ematical objects they needed, and it was too complicated
to use when it did apply. After working with Grothen-
dieck, Dieudonné concluded that Bourbaki's theory of
structures "has since been superseded by that of category
and functor, which includes it under a more general and

Early Years at Nancy. The University of Nancy made
Dieudonné an instructor in 1937 and then promoted him
to maître de conférences. He held that post until 1946,
although he was mobilized for the war in September 1939
and many university jobs were relocated to Clermont-Ferrand
when the Germans made France north of the Somme a zone interdite, forbidden to the French, under
Belgian administration. He returned to Nancy by 1943
while it was still zone interdite (Eguether, 2003, p. 25).

After the war, Dieudonné spent 1946-1948 as a professor
at the University of São Paulo, Brazil, and returned to
Nancy as a professor from 1948 to 1952. Fellow founder
of Bourbaki Jean Delsarte was then dean of the Science
Faculty and was assembling a brilliant collection of Bour-
baki members or future members at Nancy.

Delsarte and Dieudonné brought Laurent Schwartz
to Nancy. Schwartz was working on his distributions,
which made rigorous the generalized functions used by
physicists, such as the Dirac delta function. His basic tool
was to set up a relation between, roughly speaking, the
space of all real-valued smooth functions \( f \) on the real line
and the space of all generalized functions \( \varphi \) on the same
line. Exploring the foundations of his idea, he co-
authored a paper on topological vector spaces with
Dieudonné; the two of them recommended certain open
questions from that paper to their student Grothendieck.
Grothendieck in response created the idea of a nuclear
space, and Dieudonné later described his student's
answers as "the greatest advance in functional analysis
after the work of Banach" (1981, p. 220).

Work on Groups. The classical groups are certain groups
of matrices with clear geometric sense—at least, they have a
clear geometric sense when the matrices have real or com-
plex numbers as entries. For example, the real general lin-
ear group \( \text{GL}_n(\mathbb{R}) \) consists of all invertible \( n \times n \) matrices
of real numbers and is geometrically the group of all lin-
ear maps from the \( n \)-dimensional real vector space to
itself. Mathematicians since the mid-nineteenth century
had studied these, and also analogous groups with entries
in fields other than the real or complex numbers. From
the late 1940s into the 1950s Dieudonné used the rela-
tively new axiomatic theory of vector spaces, in its geo-
metrical interpretation, to simplify the proofs and clarify
the subject and solve some fundamental problems in it.

This led to Dieudonné's deepest and most imagina-
tive personal work in mathematics, his work on formal
groups. The spaces of classical algebraic geometry are
defined by polynomial equations, as for example
\( x^2 + y^2 - 1 = 0 \) defines the unit circle. Roughly speaking, a
coordinate function on the unit circle is any polynomial
\( P(x,y) \) in these same variables \( x,y \), with the proviso that
polynomials \( P(x,y) \) and \( Q(x,y) \) count as the same function
on the circle if their difference \( P(x,y) - Q(x,y) \) is a multiple
of the defining polynomial \( x^2 + y^2 - 1 = 0 \). When the polyno-
omial coefficients are taken as real or complex numbers,
then all the techniques of classical analysis apply. When
they are taken in any field \( k \), some analysis still applies
since there is a familiar formal rule for the derivative of a
polynomial. But classical techniques using limits or con-
vergent power series do not apply when the field \( k \) has no
topology (or no suitable topology) to support a notion of
convergence. This happens in particular for fields of char-
acteristic \( p \), for a prime number \( p \) where multiplication by
\( p \) counts as multiplication by \( 0 \). The algebraic geometry of
these fields was ever more central to number theory fol-
lowing work by Bourbaki members Claude Chevalley and
Weil, among others.

Dieudonné made up for a large part of the loss by
abandoning topological convergence and working for-
mally with arbitrary infinite power series. A formal group
is roughly an algebraic space where the coordinate func-
tions are not only polynomials but infinite series, and
such that the space is also a group, like the classical
groups. Through the 1950s Dieudonné made many clas-
sical analytic techniques apply in very useful ways without
using the classical notion of convergence (later collected
in Dieudonné, 1973).
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Scheme Theory. In 1952 Dieudonné accepted a one-year professorship at the University of Michigan. That led to a professorship at Northwestern University from 1953 to 1959, where he gave the lectures on analysis that became *Foundations of Modern Analysis* (1960), the final result of Bourbaki’s original plan for a textbook on analysis. This book has been a strong, immediate influence on far more mathematicians than have ever read anything else that Dieudonné wrote. It violates a stereotype of Bourbaki as it is thoroughly geometrical, but it is typical Dieudonné: It is axiomatic, quite general, and uses that generality entirely to simplify the theory. It defines derivatives as linear approximations to functions between (finite or infinite dimensional) Banach spaces and yet proves not one non-trivial theorem on Banach spaces. It uses the Banach space axioms because they assume all and only the structure needed for the basic theorems of differential calculus. They are to the point. Deep considerations on Banach spaces have no place here—simple, general facts about derivatives do.

Dieudonné left Northwestern to become the first professor of Mathematics at the Institut des Hautes Études Scientifiques (IHES) near Paris, modeled on the Institute for Advanced Study in Princeton, New Jersey. He brought his student Grothendieck, instantly making the IHES a power in mathematics. Grothendieck in fact had abandoned analysis, though, and begun the sweeping recreation of algebraic geometry around his new notion of scheme.

Roughly speaking, a scheme is an algebraic space, defined like most kinds of manifolds by coordinate functions on patches of the space, with the astonishing innovation that these “functions” need not be polynomials or even functions in any set theoretic sense. Rather, any ring in the sense of abstract algebra can be the ring of “coordinate functions” on a patch of a scheme, with the ring elements treated as coordinate functions. This appalling abstraction and generality struck many mathematicians as impervious to geometric intuition. But Grothendieck and Dieudonné knew better. They saw it not as general but as simple: The apparatus of analysis and classical topology is dropped in favor of the mere ring operations of addition and multiplication.

Much fundamental geometric intuition does survive. Notably, an algebraic triviality says that a subset $S$ of a ring $R$ generates the unit ideal if and only if some finite subset $T$ of $S$ already does. In scheme theory this has two immediate, fundamental consequences: the basic schemes are compact, and they admit analogues to Dieudonné’s partitions of unity with the difference that the “functions” $f_i$ are elements of arbitrary rings and are in no sense bounded between 0 and 1. At each point they do add up to 1.

Dieudonné took up another historic multi-volume collaboration, this time with Grothendieck as his single co-author. He did this “with the sole goal of bringing to the public the brilliant ideas of his young collaborator. One rarely sees such disinterested effort” (Cartan, 1993, p. 4). Dieudonné painstakingly organized a flood of Grothendieck’s notes into *Les Éléments de géométrie algébrique* (1960–1967), still the standard reference on schemes in the twenty-first century.

Move to Nice. In 1964 Dieudonné became the first dean of the faculty of science at the newly created University of Nice, where the Mathematics Institute is named after him. He held the deanship until 1968 and faced student unrest with his life-long respect for other people’s intentions yet rejection of all leftist politics. The professorship became honorary in 1969. He was elected to the Académie des Sciences of France in 1968, and quickly got a number of his comrades from Bourbaki into it.

He was a visiting professor at the University of Notre Dame (United States) in fall 1966 and again for two years, 1969–1971. At this time he took up his analysis textbook again and expanded to the nine-volume *Éléments d’analyse* (1968–1982). No doubt the first volume, and then the first few volumes, had more direct influence on more mathematicians than the later volumes, but the whole was a fantastic achievement and shaped the general conception of analysis for decades. The capstone of his academic career was organizing the World Congress of Mathematics in Nice in 1970. He turned to writing the history of mathematics.

In common with Weil, Dieudonné believed the history of mathematics should be impersonal, not about anecdotes, and not about priority disputes, but about the development of the leading ideas. His three key works are the historical part of his course on algebraic geometry (1974; translated to English, and expanded, 1985); his expert history of functional analysis (1981); and the massive, detailed history of twentieth-century algebraic and differential topology (1989). Any one can be recommended to graduate mathematics students learning those subjects—and any one can be faulted in detail by specialist historians. Nevertheless, they are invaluable documents. He wrote numerous entries for the *Dictionary of Scientific Biography* (Dugac, 1995, p. 119).

Tall and impressive, though not given to physical exercise, Dieudonné had an energetic enthusiasm that was punctuated by explosive bursts of temper. Friends found him optimistic, generous, honest, and with a strong sense of responsibility—although he said history precluded optimism. He distrusted political reform and was pleased to be received into the generally conservative Légion d’Honneur. He inherited strong discipline from his parents but in no ascetic way. He enjoyed good food and great wine and conversation. He was a skilled pianist and
played for an hour or two each morning. Five or six hours of sleep per night was enough.

Dieudonné called himself happy. In old age he said Socrates and Michel de Montaigne were his models for taking difficulties in the best possible way, and expressed confidence that with death, "like all animals, I will entirely disappear." He died surrounded by his wife and children, keeping the attitude he expressed five years earlier: "Now I am ready to go. If one tells me 'it will be in one month' that is perfect. I ask no more. I have had everything that I wanted in life" (quoted in Dugac, 1995, p. 20).

Honors. Among the very many honors he received, the Académie des Sciences of Paris awarded Dieudonné their Grand Prize in 1944 and the Petit D'Ormoy Prize in 1966 and made him a member in 1968. In 1966 Dieudonné received the Gaston Julia Prize. He became a correspondent of the National Academy of Sciences (United States) in 1969 and a member in 1968, at the same time as he was elected to the Académie des Sciences of France. He became a foreign member of the Real Academia de Ciencias of Spain in 1970, and of the Académie Royale de Belgique in 1974. The American Mathematical Society awarded him the Steele Prize in 1971, and the London Mathematical Society made him an honorary member in 1972. In 1978 he was made an Officer of the Légion d'Honneur.

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Dijkstra


Colin McLarty

DIJKSTRA, WYBE EDGGER (b. Rotterdam, Netherlands, 11 May 1930; d. Nuenen, Netherlands, 6 August 2002), computer science, logic, mathematics.

In 1972 Wybe Dijkstra became the first Dutch computer scientist to win the Turing Award, at the young age of forty-two. He counts as one of the founders of the discipline of computer science itself. He wrote the first Dutch textbook on programming, between 1952 and 1955. His work aimed at developing a theory of computing without computers. He was a member of the Royal Netherlands Academy of Arts and Sciences and a foreign honorary member of the American Academy of Arts and Sciences. He received a large number of prizes and distinctions.

Biographical and Career Details. Dijkstra grew up within an intellectual environment. Both his parents had taken university degrees: his mother was a mathematician, and his father was a chemist. In 1948 he finished gymnasium, the highest level of high school in the Netherlands (pupils receive education in Latin and Greek). He was groomed for a scientific career. His parents thought it would be a pity not to devote his life to science, and he followed their advice. He studied theoretical physics at one of the oldest Dutch universities, in Leiden.

In 1952 he started his career as a programmer with electronic digital computing machines. Dijkstra's entrance into the field of computing via knowledge of and experience with programming influenced Dijkstra's further career: although he participated in the logical design of some early digital electronic machines, he never involved himself with the material construction of a computer. People from all over the world joined Wilkes's courses, which means that Dijkstra belonged to the international community of computer experts from the start. It also means that Dijkstra entered the field in the context of scientific computing.

Dijkstra started working at the Center for Mathematics and Computer Science in Amsterdam (the former Mathematical Center), not yet having finished his studies. This center was subsidized by the national government, aiming at making mathematics useful for society. Numerical analysis and statistics were the core business, and this work involved a lot of computation. Therefore, the center had a computation department, which was headed by Adriaan van Wijngaarden. The first professor in computing science in the Netherlands, Van Wijngaarden was one of the key figures in the development of the computer language ALGOL 68. This computation department emerged as one of the leading institutions in the pioneering era of Dutch and European computer science in the context of scientific computing. Van Wijngaarden convinced Dijkstra to become a programmer, arguing that "computers are here to stay" (Dijkstra, EWD1308, p. 1).

In 1953 Dijkstra developed a programming manual for the first Dutch electronic digital computer, which was still being built at the time, the ARRA (Automatisch Relais Reken Apparaat, or Automatic Relay Calculator). Thus, he developed his first thoughts about programming without a machine at his disposal. Given the fact that early computers were rare, and that these were rather laboratory experiments than proper working machines, this was not so exceptional. For example, Arthur W. Burks and John von Neumann developed ideas about coding in 1946-1947, before the IAS (Institute for Advanced Study) machine had been built.

Dijkstra wrote, together with Van Wijngaarden, the first programming textbook, which was well entrenched in current knowledge and practice of the early 1950s. It included a discussion of the computer itself (the ARRA), a section on flowcharting, a library of subroutines, examples of programs, and a discussion of interpretive programming. In this textbook, one chapter is fully devoted to reliability of the results. This is a topic that Dijkstra continued working on throughout his career, especially with regard to software.

Professionalization of Programming. Maurice Wilkes built one of the first electronic digital stored-program computers in the world, the EDSAC (Electronic Delay Storage Automatic Calculator) at the University of Cam-

Mac Lane in his long life made powerful and lasting contributions to world mathematics, notably by crystallizing with Samuel Eilenberg the concepts of category, functor, and natural transformation, and then extensively developing and applying them. Those concepts have become indispensable to twentieth- and twenty-first-century thinking about geometry, algebra, and logic and have a growing simplifying influence on analysis, statistics, and physics. One of the few universal mathematicians, Mac Lane was a towering figure because of his enormous work in research and teaching, but his career was also marked by a persistent struggle to bring about change and the acceptance of new ideas through participating in organizations whose tendency was rather to uphold the status quo. Mac Lane’s doctoral students, including Irving Kaplansky, John Thompson, Michael Morley, and Robert Solovay, played important roles in twentieth century mathematical research. He was very active in the Mathematical Association of America, the American Mathematical Society, the National Academy of Sciences (NAS), and in the International Mathematical Union. He received numerous prestigious prizes for scientific achievement, including the National Medal of Science in 1989; in 1972 he was named an Honorary Fellow of the Royal Society of Edinburgh.

Early Life and Career. Mac Lane was 15 years old when his father died and he went to live with his grandfather. His father and grandfather were both pastors in the Congregationalist Church, and Mac Lane admired both of them for their courage in preaching nonconformist views such as Darwinism and pacifism, but he could never accept their religious ideas. An uncle financed his study at Yale University, where he graduated in 1930 with the highest academic standing in the history of the university, but he was not elected to the notorious Skull and Bones. He earned his master’s degree in 1931 at the University of Chicago, where Eliakim H. Moore counseled him to go to Germany; he became the last American to earn a mathematics doctorate at the University of Göttingen of David Hilbert, Emmy Noether, Hermann Weyl, and Paul Bernays. He hid his copy of Das Kapital to prevent its being burned when he witnessed the Nazis taking control in January 1933. After his return to the United States, he taught mathematics at Cornell, the University of Chicago, and for ten years at Harvard University. In 1947 he became a professor at the University of Chicago, where his vigorous and inspired teaching continued well after his retirement in 1982.

Activities in Professional Organizations. From 1943 to 1981 Mac Lane was very active in various professional organizations. He served as director of Applied Mathematics at Columbia University from 1943–1945, on leave from Harvard, as part of the war effort. He was president of the Mathematical Association of America in 1951–1952, and began efforts at the national level to reform the teaching of mathematics. On 1 February 1952 he issued the directive to all sections of the MAA that minorities must have equal access to the academic and social functions of the association, contrary to the
previous practice of some of the sections. From 1952–1958 he was chairman of the mathematics department at the University of Chicago, succeeding his friend Marshall Stone. In 1973–1974 he was president of the American Mathematical Society and was vice president of the National Academy of Sciences until 1981. During the eight years at the NAS he devoted much of his energy to chairing the Reports Review Committee, and was from time to time compelled to issue forceful calls for greater scientific seriousness—for example, in connection with a report on the effects of the military use of poisons in Vietnam. He did the same in other contexts, which, as one would expect, led to a mixed popularity.

Mac Lane strove valiantly to promote that closer unity between teaching and research that was so much the essence of his own mathematical life. To advance that purpose, he urged a merger of the professional societies, but succeeded only in creating one umbrella committee, the Joint Policy Board for Mathematics.

Through his organizational initiatives at the national level in the mid-1950s, he had applied his international mathematics experience to courses for high school teachers, which he and other active mathematicians taught. In the early 1960s, however, hopes for a progressive new math were frustrated when university presidents and government agencies cut the funding for these courses. Channelling energies into a retrogressive “new math,” various authorities made organizational decisions that, in Mac Lane’s view, tended to steer high school teachers towards outmoded pedagogical theories, instead of scientific thinking and mathematical content. Mac Lane’s efforts to promote improved conditions for scientific research and education achieved only modest results, in spite of the great amount of time he spent in Washington. That experience contributed to his later analysis of what he saw as grave flaws in the methods for arriving at science policy in the federal government and the American university system.

Influential Textbooks. Fortunately, Mac Lane’s energies were not entirely devoted to organizational efforts, but also to his own fruitful research and teaching and especially to the relation between them. His textbooks A Survey of Modern Algebra (1941, with Garrett Birkhoff), Homology (1963), Categories for the Working Mathematician (1971), and Sheaves in Geometry and Logic (1991, with Ieke Moerdijk) are still widely used in the early twenty-first century. Mac Lane’s book with Birkhoff made Bartel van der Waerden’s Moderne Algebra (1930) accessible to English-speaking undergraduates. The Survey was fundamental to the education of several generations, and Mac Lane rewrote the 1967 edition in order to respond explicitly to the growing need for the learning of category theory. All four of these textbooks fundamentally contributed to bringing new abstract research to students at the time when they needed to learn it.

Homological Functors and Abelian Categories. Forceful personality and energetic perseverance were not the only attributes that made Mac Lane so prominent; rather, the primary reason is that his central ideas were, and have remained, correct. He accurately summed up the achievements of the previous generation and passed them on, forever transformed in clarity and applicability. This process can be clearly discerned in the cases of homological functors and Abelian categories.

By 1940 Mac Lane and his friend Sammy Eilenberg had each made significant contributions to their respective fields of algebra and topology, and thus, through their collaboration on the challenging problems of Heinz Hopf and Norman Steenrod in algebraic topology, could gain access to the rich social patrimony of several centuries of mathematical development. Reflected through that access was sufficient knowledge of the forms of results, and especially of some main modes of the development of ideas, so that they were able to concentrate and isolate the explicit concepts that they called category, functor, and natural transformation. These explicit concepts were so correct as a reflection of the essence of various aspects of mathematical content and motion that they immediately provided a source of structures whose properties could be studied with fruitful results for mathematics in general. The concept of functor, almost immediately after its discovery by Eilenberg and Mac Lane in 1942 (and expounded by them in 1945) provided a structure to which axioms and deduced theorems could be applied; specifically, the axioms announced by Eilenberg and Steenrod in 1945 (and expounded by them in 1952) clarified the previous proliferation of geometrical constructions known as homology theories, and in turn made possible still richer such theories. Functorial homology theory became a cornerstone of the still ongoing research in algebraic topology. The axiomatic method could similarly be applied to categories themselves, as was then exploited by Mac Lane (in 1948, expounded in 1950). He captured the essence of linear algebra via the axiom that products and coproducts (which themselves can only sensibly be defined by categorical means) coincide in certain categories. Often such categories enjoy the internal representability of the solutions of any equation they contain; if they satisfy certain “exactness” conditions, such categories are called “Abelian” after the great Norwegian mathematician Niels Abel (1802–1829).

These Abelian categories quickly served their purpose as another cornerstone of algebraic topology and were host to a new branch of linear algebra that became known as homological algebra. Homological algebra had
undergone extensive development in the collaboration of Eilenberg and Mac Lane in the late 1940s and early 1950s on the homology of groups. Over the next decades, Abelian categories underwent deep development by David Buchsbaum, Alexander Grothendieck, and Jean-Pierre Serre, and further by Maurice Auslander, Michael Barr, Peter Freyd, Peter Gabriel, Alex Heller, Barry Mitchell, Stephen Schanuel, Jean-Louis Verdier, Nobuo Yoneda, and others. Philosophically, those developments meant in particular that methods previously conceived as applying only to constant quantities could be extended to apply also to variable quantities, with powerful results.

**Adjoint Functors.** Axiomatic algebraic topology and homological algebra can both be described as having been a new category theory, arising in a sense entirely within category theory, but in response to the needs of application. The most important instance of this phenomenon, it is generally agreed, is Daniel Kan’s 1958 discovery of adjoint functors (which, in retrospect, were implicit in the Eilenberg-Mac Lane 1945 paper). This concept united a wealth of old and new examples, again exploiting the susceptibility of the appropriate structures to restricting properties that, as axioms, have powerful consequences and serve as a guide to further constructions, conjectures, and theorems. The particular problems occupying Kan concern the relation between the qualities of combinatorial homotopy theory and the qualities of quantities arising in differential vector calculus (a relation that lies at the basis of the finite element method in applied electromagnetism, for example). Kan discovered that a functor from one category to another might be so special as to have another uniquely determined functor in the opposite direction that, while not actually inverting it, is the “best” approximation to an inverse (in either a left- or a right-handed sense). Typically, one of the two functors is so obvious that one might not have mentioned it, whereas its resulting adjoint functor is a construction bristling with content that moves mathematics forward.

One of many examples, whereby the use of adjoint functors helps old constructions become much more explicit and clear, is a construction which had played a key role in Mac Lane’s research in Galois theory and explains the realization of conjugation on the complex numbers (the process of negating the generator i) as an inner automorphism (by j) of the larger enveloping algebra of Hamilton’s quaternions. There is a similar realization of the mechanical flow on a phase space via an inner derivation by a Hamiltonian element in an algebra of operators. The conjugation example involves a two-element group in two different roles, and the mechanical example similarly involves an infinitesimal group of time translations. The “inner realization” construction is the left adjoint of the functor, determined by a given group G, which to every algebra A with a given representation of G by multiplication in A, assigns the action of G on A defined by

\[ ag = g^{-1}ag. \]

The left adjoint to this process applies to any algebra with a given action of G on it, enlarging it in an optimal way to make the action inner; the resulting algebra, which contains this inner action of G in it, is usually non-commutative, even if the given one was commutative.

**Adjointness and Logic.** Naturally, Kan’s discovery spurred a succession of new leaps forward within category theory in response to its relation with applications. Some of these were intimately related with Mac Lane’s longstanding interest in logic and set theory. Already during the 1931 interval between his Yale degree and his studies at Göttingen, he had studied at the University of Chicago with Eliezer H. Moore. Moore too was a very strong personality who, coming from algebra, had, like Mac Lane, a burning desire and specific proposals to unify mathematical research and to reform mathematical teaching to that end. Mac Lane had taken up from Moore the quest to axiomatize set theory. Mac Lane’s thesis at Göttingen had resulted from intense discussions with the set theorist Paul Bernays concerning the possibility of a formalized logic that could actually be used to guide mathematical proofs. During the next 30 years, however, Mac Lane did not concentrate his research on set theory or logic, although he did make valiant organizational efforts, promoting the formation of logic clubs among undergraduates and reviewing for the *Journal of Symbolic Logic.* He was pleased in the early 1960s when it became apparent that logic and set theory, insofar as they are mathematically relevant, can be characterized axiomatically as specific interlocking systems of adjoint functors: specifically

1. propositional logic symbolically presents parts of a universe of discourse in terms of pairs of operations like \( G \& (\cdot) \) and \( G \) implies \( (\cdot) \), related by rules of modus ponens and deduction which say no more than that those operations are adjoint;
2. predicate logic treats, moreover, parts of several universes related by maps (for example projection maps), where the fundamental categorical process of composition is exemplified by substitution along the map (representing inverse image) that has a left adjoint, namely existential quantification along the map (representing direct image); and
3. higher-order logic treats, moreover, a system of several universes wherein, for any universe \( G \), there are adjoint functors creating as new universes the \( G \)-cylinder and \( G \)-figure universes (also known as...
function types; part of the adjunction property had been called lambda-conversion by Alonzo Church).

Set theory itself was quickly seen in a new light via adjointness; after all, the adjointness of function types expressed a fundamental transformation that had been used in functional analysis (and in its embryo, calculus of variations) for 250 years, and the belief that Georg Cantor’s set theory would have an important role in analysis (as expressed at the first International Congress of Mathematicians in 1897 by Jacques Hadamard) had sprung from the intense work at that time which was bringing functional analysis to the light of day. Quickly overcoming his initial skepticism, Mac Lane recognized the decisive importance, for set theory and logic and their relationship to mathematics, of the explicitly adjoint character of these operations. He sprang into action: He made sure that the basics were published by making F. William Lawvere’s *Elementary Theory of the Category of Sets* available through the University of Chicago library, and ensuring that an announcement of that work appeared in the *Proceedings of the National Academy of Sciences* (1964); he wrote expositions himself for all kinds of audiences; and he engaged in published polemics with recalcitrant set-theorists, right up to the new millennium (2000).

**The Geometrical Use of Category Theory Gives Rise to New Category Theory.** During the same period, Alexander Grothendieck was creating the new foundation for algebraic geometry, which was also based on categories and adjoint functors. He realized that the variable linear algebra that he (following Mac Lane) had developed in the late 1950s, is best viewed as an additional structure on a non-linear kind of category, such as set-valued sheaves or analytic spaces. This led to the crystallization of a new kind of categories, which he called toposes because they were the brave new manifestation of the science of situation. (The Greek term was apparently chosen to signify a qualitative deepening of the analysis situs of Henri Poincaré.) These situations serve as the domains of variation for variable sets. In the 1970s students in Paris, at Harvard, and at other centers had to struggle to learn the topos theory through the 2000 pages (written with the help of Michael Artin, Jean-Louis Verdier, and others) that only the brain of a Grothendieck could really encompass.

Grothendieck retired suddenly in 1970, but the new algebraic geometry continued to develop. Meanwhile a simplified form of the topos theory had already sprung up, with motivations from continuum mechanics, but with new applications to set theory and logic. (That work, achieved in collaboration with Myles Tierney, was presented in 1970 by Lawvere at the International Congress of Mathematicians in Nice, France.) For a time those two trends, algebraic geometry and the new topos theory, were very slow in learning from one another, in contrast with the situation in the 1950s when category theory had been revolutionizing not only the framework of mathematics, but also its practice. There were logicians who, like Galileo’s colleague, refused to look into “the telescope” that was provided by books on the topos-theoretic simplification of logic; there were also algebraic geometers who dismissed the modern topos geometry as “mere logic.” Mac Lane did not despair in the face of these difficulties of communication and lack of mutual understanding.

That was precisely the sort of wrong that Mac Lane knew how to begin to set right: girding himself anew in the middle of his seventh decade, he set off for Cambridge where his lectures inspired Peter Johnstone to write the first book on the new topos theory (1977). In the middle of Mac Lane’s ninth decade, yet another book appeared, the result of his collaboration with Ieke Moerdijk of the University of Utrecht in the Netherlands; this latter book, *Sheaves in Geometry and Logic* (1992), was necessary because the books that had appeared in the intervening twenty years on this emerging subject had still not covered all the varied developments. The book with Moerdijk shows clear traces of the hand of the master expositor: the systematic use of presheaf toposes and Kan adjoints and the careful exposition of the relation between the combinatorial topology of 1950 and the internal role of the adjointness formulation of logic.

**Continuing Influence.** Algebraic geometry, complex analysis, universal algebra, logic, and in fact any given field in mathematics has as neighbors other growing fields wherein the categorical method is already indispensable. Homotopy theory, for example, uses the Eilenberg-Mac Lane spaces (introduced in the mid-1940s) and is often based on Daniel Quillen’s axioms, built on the categorical work of Peter Gabriel and Michel Zisman. As another example, the closed and enriched categories of Samuel Eilenberg and Gregory M. Kelly have led to a rejuvenated study of metric spaces and of generalized logic, unexpected when they were introduced in 1965 as the definitive solution to the problem of signs in algebraic topology. The force of the streams of these neighboring developments will eat away any resistance of remaining “islands” to the unification that the progress of mathematics requires.

When his formulation of the homology of rings was taken up and developed in 1961 by Umeshchandra Shukla, Mac Lane remarked that “as always, it is a pleasure to see how new ideas spread” (2005, p. 222). When visiting Soviet Georgia in 1987, he found to his delight a group of a dozen devoted and enthusiastic researchers in category theory and concluded, “it is remarkable to see
how abstract mathematical ideas have international resonance" (2005, p. 331).

The enormous power of a correct idea cannot always be foreseen. Mac Lane found it hard to believe that already by the late 1950s, the fundamentals of category theory had penetrated into the midwestern farmlands, enabling fledgling students there to discuss and dream about its intimations of powerful unifying developments. The correctness of these explicit ideas was an electrifying inspiration to us then, and remains an enduring inspiration to scientific progress.

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MAIMONIDES, RABBI MOSES BEN MAIMON, also known by the acronym RaMaB (b. Córdoba, Spain, 1135 or 1138; d. Cairo, Egypt, 1204), medicine, codification of the Jewish law, Philosophy. For the original article on Maimonides see *DSB,* vol. 9.

Maimonides is widely held to be the most important Jewish philosopher of the premodern period, perhaps of all times. His accomplishments in diverse branches of science, most especially medicine and astronomy, have contributed very much to the development of this attitude. The authority of Maimonides’ thoughts on the relationship between science and religion are hugely enhanced by his eminence as a scientist. Maimonidean studies have burgoned in the decades since the appearance of the first edition of *DSB,* and, along with this, his activity and literary legacy in the sciences have been closely scrutinized.

Interpolation of Maimonides’s Epistemology. No aspect of Maimonides’s involvement in the sciences has generated as much interest as the precise determination of the limits he placed on human knowledge, particularly with regard to the physical configuration of the heavens. A fundamental principle of cosmology, allegedly tracing back to Plato, stated that the motions of heavenly bodies are circular and centered upon Earth. In order to account for the apparent anomalies from uniform circular motion, astronomers employed devices such as eccenters and epicycles, which violate this principle. Maimonides was one of several prominent medieval thinkers who possessed a firm grounding in mathematical astronomy and who were deeply troubled that the models they used were not in keeping with the rules. Maimonides concluded his review of the difficulties besetting the astronomy of his day (*Guide of the Perplexed* 2.24), for which he could find no acceptable solution, with two contradictory remarks. First, he suggested that one cease speculations concerning things that are beyond the intellectual capacity of regular human beings. Immediately afterward, however, he

Weil was an arithmetician in the broadest possible sense. His work on Diophantine equations drew on all the fields of pure mathematics and developed methods so deep and elegant as to influence each of those fields in turn. He was a founder of the Bourbaki group and its strongest mathematician, the most prominent mathematician at the University of Chicago when it was arguably the world’s leading mathematics department, and a member of the Institute for Advanced Study. He decisively shaped the style and direction of all post–World War II mathematics.

Childhood. The Alsatian Jewish medical doctor Bernard Weil and Russian–Austrian Jewish Salomea (Selma) Reinherz Weil were very comfortably established in Paris when their first child, André, was born, and three years later their daughter Simone, who became known as a philosopher. The children, raised especially by their mother, were precocious and accomplished. The parents had seen bitter anti-Semitism in eastern Europe and in the Dreyfus affair in France. They raised their children so thoroughly assimilated that André was around thirteen before he learned that Jewish descent could matter in any way. At age eight, as a gift to their father, André taught his five-year-old sister to read the newspaper aloud to the family. By age twelve he worked on university-level mathematics, played the violin, taught himself to read Homer and Plato in Greek, and to read Sanskrit. The family often conversed in English or German.

At age fourteen, three years below the minimum age, he took the state baccalaureate exam by special permission and got the highest scores in the nation. He began preparing for the exam to enter the École Normale Supérieure (ENS), which generally takes two years and not rarely more. He took one. During that year with advice from Jacques Hadamard he began to study analysis and differential geometry.

He entered the ENS with the highest exam scores in the nation. He felt he became a mathematician in Hadamard’s nearby seminar at the Collège de France, where he presented at least once: on domains of convergence of power series in several complex variables. He took courses with Henri-Léon Lebesgue and Charles-Émile Picard, which did not prevent his also studying Sanskrit at the Sorbonne and reading the Bhagavad Gita in the original, which he would carry with him for the rest of his life both for its poetic beauty and its philosophy.

Beginning a Career. During his time at ENS he lived at home as did many students with family in Paris. The family was extremely close, and brother and sister were devoted to each other. He graduated from the ENS at nineteen, too young for military service, and so had time for what he later regarded as his gift for traveling. A summer with his family in the French Alps left him with notebooks full of Diophantine equations plus a plan to always write so as to draw the reader beyond the manifest content toward yet more distant perspectives.

A scholarship from the Sorbonne took him to Rome for six months of mathematics and study of Italian painting up through the modernists. He heard Francesco Saverio on algebraic surfaces and encountered Solomon Lefschetz. He read a paper containing a theorem and a conjecture by Louis Mordell (1922) without guessing how important they would soon be to him. Support from the Rockefeller Foundation let him spend much of 1927 in Göttingen, Germany. He describes encounters with Richard Courant, Emmy Noether, and others in his autobiography (1991). Over this time and the next year he crystallized a thesis topic based on the Mordell paper.

Weil was steeped in the long-prestigious subject, from Niels Abel and Carl Jacobi at the beginning of the nineteenth century to Karl Weierstrass and Jules-Henri Poincaré at the end, of integrals of multiple-valued complex functions or in modern terms integration on Riemann surfaces. His teachers Hadamard and Picard were personally involved in it. An elliptic curve, or genus one Riemann surface $C$, is topologically a torus and algebraically is defined by a nice cubic polynomial $P(X,Y)$ in two variables. Integration along paths on $C$ produces a natural Abelian group structure on the points of $C$. Any points $p,q$ of $C$ have a kind of geometrical sum $p+q$ and this addition law is associative, commutative, and has a zero point and additive inverses. This group structure efficiently organizes the theory of integration on $C$. Higher degree polynomials $P(X,Y)$ define Riemann surfaces of higher genus, which topologically are surfaces with more than one torus-handle.

The number of handles $g$ is called the genus of the surface. Jacobi already knew in effect that integration on a genus $g$ surface $C$ is organized by a group structure on a space $\mathcal{J}(C)$ of complex dimension $g$, called the Jacobian of $C$. A genus one Riemann surface is its own Jacobian. A higher genus Riemann surface $C$ maps as a complex 1-dimensional subspace into its complex $g$-dimensional Jacobian $C \to \mathcal{J}(C)$.

Poincaré drew arithmetic conclusions from the trivial observation that if a cubic polynomial $P(X,Y)$ has rational coefficients then any geometrical sum $p+q$ of rational points on its curve $C$ is again rational. He conjectured that the group of rational points is finitely generated: a finite number of rational solutions to the cubic $P(X,Y)$ suffices to generate all the rational solutions by the addition law. Mordell proved this and gave his own conjecture: in genus
higher than one a Riemann surface \( C \) has at most finitely many rational points. Weil set out to generalize Mordell's proof, prove Mordell's conjecture, and introduce systematically useful tools to do it. He succeeded at the first goal and the last.

In place of rational points Weil proved the theorem for points in any algebraic number field \( k \); that is: Fix any finite list of irrational numbers, and take solutions using those irrationals along with the rational numbers. And he proved it for the Jacobian of any Riemann surface: For any Riemann surface of any genus, defined over any algebraic number field \( k \), the group of \( k \)-valued points of the Jacobian is finitely generated. Weil's clear organization of the proof made the more general conclusion natural. His proof presaged the arithmetic theory of heights, where rational numbers are ordered by increasing complexity so that there are only finitely many below any fixed level of complexity. This allows inductive proofs on the complexity of rational points. And he made elegant use of Galois groups, presaging Galois cohomology bringing methods of algebraic topology into arithmetic. Typical of Weil's work, it is a tough, elegant argument and inspired much further progress.

Hadamard encouraged Weil to prove the Mordell conjecture in his thesis so as not to leave the work half done. Weil's result on Jacobians suggested a strategy: The rational points (or \( k \)-valued points) of \( J(O) \) are finitely generated and so are sparse in \( J(C) \). For genus \( g \) greater than one, the surface \( C \) forms a 1-dimensional subspace of its higher dimensional Jacobian \( J(O) \), thus also sparse. Two sparse subsets should meet only rarely—and the right details might show these meet only finitely many times. A proof would have to be much more sophisticated and no one has yet made it work. The theorem was proved fifty-five years later by Gerd Faltings (1983), by quite other means descended from the Weil conjectures, described below. Weil liked to say he had done well to reject Hadamard's advice and submit his dissertation as it was.

Weil lived with his family as he wrote the dissertation and indeed during his following year of military service. Because of his age he missed the military training usual at the ENS. So he was placed in the infantry rather than the standard artillery, and officials secured him an easy station in Paris. He got leave time to correct the printer's proofs of his dissertation.

Having made clear his desire to see India, Weil was offered a job at Aligarh Muslim University near Delhi. He agreed to teach French civilization but then the university could not create a position for it. He would have to teach mathematics. He did so from 1930 to 1932. Back in France he was highly esteemed by top mathematicians, but arithmetic was an odd specialty there at the time. Few could read his dissertation. He found a good position at the University of Strasbourg and held it until 1939.

During these years he worked in analysis, especially integration on topological groups, and on algebraic and arithmetic topics derived from his dissertation. His most widely used innovation was in point set topology, namely the idea of uniform spaces. Such a space has no metric giving a distance between points, yet it makes sense to talk of different sequences "converging at the same rate" to different points. In particular there is a well-defined notion of uniform convergence of a series of functions from one uniform space to another.

Bourbaki. France could claim to lead nineteenth-century mathematics. Germany had a decisive lead by the 1930s in part because, unlike France, Germany had a policy of protecting promising academics through World War I. Weil happily visited Germany but was ambitious for his own country. Many young mathematicians in France were unhappy with their outdated curriculum. And classmates from the ENS by design looked to each other as an elite destined precisely to assure French greatness in all things. In 1934 Weil assembled a handful of his friends, all admitted to the ENS between 1922 and 1926, to write a definitive new analysis textbook. He met with Henri Cartan, Claude Chevalley, Jean Delsarte, Jean Dieudonné, and René de Possel at the now-vanished Café Capoulade near the ENS on Monday 10 December 1934. The textbook project expanded into a series of books covering the basics of all pure mathematics, none of which would make any references except to earlier books in the series. The group published under the name Nicolas Bourbaki and kept enough secrecy that for decades many mathematicians were unsure just who this Bourbaki was.

These mathematicians had closely similar backgrounds, tastes, and goals. The work was intensely collaborative and cannot be divided into parts attributable to each separate member. But nothing came out of Bourbaki against Weil's wishes. Indeed nothing came out at all for several years. Weil spent much of 1937 at the Institute for Advanced Study in Princeton, New Jersey. He returned to France via New Orleans and Mexico. Soon after that the book series was named the Éléments de Mathématique. The French title Éléments de mathématique expresses the unity of mathematics by using a made-up singular noun mathématique rather than the standard plural form mathématiques.

The first volume appeared in 1939. It was a preliminary treatment of set theory and the basic idea of structure. By the 1950s Bourbaki produced books on algebra, topology, functions of one real variable, topological vector spaces, and integration—very close to Weil's range of research topics. Other volumes came later and the
Elements of Mathematics were never actually completed, but the series had a huge influence on worldwide standards for rigor and style of argument. Bourbaki became the standard reference fixing the definitions of modern terminology in most fields of mathematics. Perhaps the main influence was to reorganize all of pure mathematics around recent abstract techniques instead of traditional subject areas. Weil was the leading member of Bourbaki until, following a rule that he had introduced at the beginning, he retired from the group at age fifty.

World War II and America. During the early Bourbaki years Weil met Évelyne (Eveline) de Possel, then wife of René de Possel, who divorced Possel and in October of 1937 married Weil. They would have two children, daughters, Sylvie born 12 September 1942 and Nicolette born 6 December 1946, after they left Europe for the United States.

As World War II approached, Weil thought of the philosophy of the Bhagavad Gita and of the loss France suffered in World War I by not protecting her scientists. He rejected the general pacifism of his sister—as she would also during the war—but resolved that if war came he had a duty to keep himself out of it by going to the United States. When it came he was in Finland with that plan in mind. He stayed there until he was arrested as a suspicious foreigner. The story that he was nearly shot as a spy seems to be exaggerated (Pekonen, 1992). He was shipped to jails in Sweden, England, and finally France, where he was arrested for failing to report for military duty. Convicted in May 1940, he asked to be sent to the front rather than jail, and this was granted. The front collapsed before he reached it. In January 1941 he, Eveline, and his parents left for the United States.

The Rockefeller Foundation helped him get teaching work briefly at Haverford College. There he began influential work in geometry with all of his hallmarks: a classical problem with many easily visualized cases is elegantly solved and generalized by using the latest reputedly abstract tools. Karl Friedrich Gauss showed that the sum of the angles of a triangle on a hyperbolic (constant negative curvature) plane is less than 180°, while the angle sum on an elliptic plane (constant positive curvature) is greater than 180°. Much more, on either kind of plane, the difference from 180° is directly proportional to the area of the triangle. The Gauss-Bonnet theorem generalized this to any region $P$ surrounded by a curve $C$ on a surface $S$ of variable curvature, where $P$ itself may have some complicated topology. The integral of the surface curvature over $P$ replaces the area of the triangle, while the integral of the geodesic curvature along $C$ replaces the angle sum. The sum of the two integrals equals $2\pi$ times the Euler number of $P$ which measures how many "holes" and "handles" the region $P$ has. In particular the integral of the curvature over an entire surface $S$ is always $2\pi$ times some integer uniquely determined by the topology of $S$. Carl Allendoerfer at Haverford generalized this latter result to $n$-dimensional manifolds $M$ embedded in some Euclidean space $\mathbb{R}^n$. Allendoerfer and Weil (1943) together generalized the whole Gauss-Bonnet theorem to $n$-dimensional regions $P$ in Riemannian manifolds and mistakenly thought they had eliminated the need for a Euclidean space.

The next year Shing-Shen Chern, visiting the Institute for Advanced Study in Princeton from China, sharply simplified the Allendoerfer-Weil proof and did eliminate the Euclidean embeddings. He used a very pretty geometric construction with a fibre bundle over the manifold $M$, that is a map of manifolds $B \to M$ which in Chern's case bundles together one $n$-1 dimensional sphere for each point $x$ of $M$ and depicts each as the sphere of unit tangent vectors at $x$. Fibre bundle techniques were new and growing quickly at Princeton. A series of private letters by Weil used all of the tools of integration on manifolds, and topological groups, and cohomology to streamline Chern's construction. Typical of his best work, Weil showed how a quick and natural treatment of that construction led seamlessly to a vast generalization. The Chern-Weil homomorphism gives an analogue of Gauss-Bonnet for any fibre bundle with a connection, that is with an abstract analogue of differentiation along tangent vectors. The abstract concept has geometric uses far remote from the original motivation. The result and the means used to prove it became cornerstones of the theory of characteristic classes on fibre bundles.

Weil next worked at Lehigh University in Pennsylvania. Depressed by the heavy teaching load and uninterested students, in 1944 he resolved to quit and do anything else. The structural anthropologist Claude Lévi-Strauss got him a position at the Universidade de São Paulo in Brazil, a center for algebraic geometry. With a visit to Paris in 1945 he stayed at São Paulo until 1947, when he was appointed professor at the University of Chicago. He had a leave in Paris for 1957–1958 and then became a professor at the Institute for Advanced Study starting in 1958 and retiring in 1976. He traveled back to India in 1967, and made several trips to Japan.

Simone joined Charles de Gaulle's Free French movement in England. She had adopted a passionate Christian asceticism and self-denial, she requested to be sent on hopelessly dangerous missions in France, and ate less and less, purportedly to express solidarity with the suffering in France. Stricken with tuberculosis, she refused food and medical care. She died 24 August 1943. André was devastated by the loss and by her role in it. He helped produce her posthumous publications and never got over
Weil

The Weil Conjectures. Nineteenth-century number theorists already saw deep analogies between the ordinary integers \( Z \) on one hand and polynomials in one variable with complex coefficients \( \mathcal{O}(z) \) on the other. The square root of 2 is an algebraic number because it satisfies an integer polynomial equation namely \( X^2 - 2 = 0 \) so that \( X = \sqrt{2} \). The complex square root function is an algebraic function since it satisfies a similar equation \( X^2 - z = 0 \) so that \( X = \sqrt{z} \) where \( z \) is not a number but a variable over the complex numbers. Theorems on algebraic numbers often had analogues for complex algebraic functions although the proofs might be quite different. Sometimes the proof was easier for algebraic numbers and sometimes for algebraic functions. No general, routine way of turning proofs for one case into proofs for the other is known as of 2007.

Weil wrote to his sister about this, saying, “we would be badly blocked” if we could find no good link between the cases, but “God beats the devil” because there is a promising intermediary: replace the complex numbers by any finite field (Letter of 26 March 1940, in Weil, 1979, vol. 1, p. 252). One simple finite field is the integers modulo 5, which is a five-element set \( \{0, 1, 2, 3, 4\} \) with addition and multiplication defined by casting out 5s. So \( 1 + 2 = 3 \) as usual, but \( 3 + 4 = 2 \) modulo 5. This is obviously like the ordinary integers, but also like the complex numbers in the key respect that each integer has an inverse modulo 5: \( 1 \times 1 = 2 \times 3 = 4 \times 4 = 1 \) modulo 5.

Weil proved an analogue to no small theorem for this case, but to the deepest most sought-after theorem in number theory: the Riemann hypothesis. In 1940–1941 he proved a Riemann hypothesis for curves over any finite field.

These curves have genus in the same algebraic sense as Riemann surfaces although they are not continuous curves or surfaces in any way, so their genus has no evident topological meaning. The “coordinates” of points on such a curve lie in finite fields rather than in the continuous complex plane. Yet Weil saw that great results would fall out if he could generalize topological ideas related to the genus of continuous curves.

During Weil’s time in Göttingen he heard of Heinz Hopf’s work on the Lefschetz fixed point theorem using topology to count the solutions to suitable equations without actually solving them. Weil made pioneering use of the theorem in 1935 in an elegant new proof of a known theorem on Lie groups. By the late 1930s the relevant topological tools were embodied in cohomology theory. He found an often-useful method of showing when different cohomology theories will agree on specified cases. But the known cohomology theories all relied on the continuity of the real numbers. They had no contact with finite fields or in other words with finite characteristic. Yet Weil conjectured that an analogue in finite characteristic could give stunningly simple proofs of some powerful arithmetic claims.

The conjectures became famous as the Weil conjectures. They are concise, harmonious, penetrating, and surprising. The known special cases were already impressive. They were too beautiful not to be true. Yet it was nearly inconceivable that they could even be stated precisely. Weil himself would not affirm that such a cohomology theory could exist. Perhaps he did not believe it could. Jean-Pierre Serre did, and convinced Alexander Grothendieck, and took part with him and Pierre Deligne in creating it: “This truly revolutionary idea thrilled the mathematicians of the time, as I can testify at first hand; it has been the origin of a major part of the progress in algebraic geometry since that date. The objective was reached only after about twenty-five years, and then not by Weil himself but (principally) by Grothendieck and Deligne” (Serre, 1999, p. 525).

It is typical of Weil that his sweeping vision of unity among the great branches of mathematics produced specific insights and proved hard theorems in specific branches. In this case the impact began with algebraic geometry and number theory. It required a thorough reconception of topological tools in explicit algebraic terms and these fed back into algebra and topology. It affected the related parts of complex analysis and differential geometry. It had eventual repercussions for more or less all of mathematics. These conjectures were Weil’s single most influential and ultimately most productive contribution. No doubt he hoped and intended to prove them as well, but the conjectures as such were a work of transcendent genius, and he knew it.

Weil also contributed to the so-called Taniyama-Shimura-Weil conjecture that all elliptic curves are modular. But this might better have been called a question than a conjecture. None of the three felt strongly that it was true. Weil never claimed he originated it. He worked on it and encouraged work on it. The key advance on it came when Andrew Wiles, using tools descended from the Weil conjectures proper, proved nearly the entire Taniyama-Shimura-Weil conjecture as the last step in the proof of Fermat’s Last Theorem.

He kept residence in Princeton after retiring in 1976. But he spent each spring in Paris in his parents’ apartment overlooking the Jardin du Luxemburg. He spent each summer in the Mayenne where Brittany meets Normandy and the Loire Valley. He wrote history of mathematics in a way that affected current research in number theory,
especially his 1976 book on Eisenstein and Kronecker. His wife’s death led him to write his autobiography (1991). By age eighty he suffered poor eyesight and failing health and he died of old age while his formidable and sometimes mocking personality left him feeling isolated from colleagues. Despite his attachment to the Bhagavad Gita he expected no personal survival after death. He was confident that the work of Bourbaki would endure. In the early twenty-first century that work is less in fashion. Yet it remains a decisive influence on mathematics.

Honors. Weil became an honorary member of the London Mathematical Society in 1959, a Foreign Member of the Royal Society of London in 1966, a member of the U.S. National Academy of Sciences in 1977, and of the French Académie des Sciences in 1982. He received the Wolf Prize in Mathematics for 1979 jointly with Jean Leray, and was presented the Barnard Medal by Columbia University in 1980, the AMS Steele Prize of 1980 for lifetime achievement, and the Kyoto Prize in 1994.

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WEISMANN, AUGUST FRIEDRICH LEOPOLD (b. Frankfurt am Main, Germany, 17 January 1834; d. Freiburg im Breisgau, Germany, 5 November 1914), zoology, heredity, evolution. For the original article on Weismann, see DSB, vol. 14.

The necessary starting point for evaluating Weismann’s career remains his zoological writings. This is not a large corpus in comparison to those of some of his contemporaries, say, Charles Darwin or Ernst Haeckel. It consists, however, of professionally solid and challenging monographs and books, including Das Keimplasma (1892), that were influential in their own day and still referred to in the early twenty-first century. It also consists of many elegant essays and monographs on evolution and heredity. Finally, there exist three editions of Weismann’s comprehensive advanced textbook, Vorträge über Descen­denztheorie (1902, 1904, 1913) devoted to a neo-Darwinian view of evolution. This comprised the first such modern textbook that framed an elaborate mechanistic model for evolution, heredity, and development. Most, but not all, of this literature was translated into English during Weismann’s lifetime.