MATH 435 – Fall 2016
Syllabus and Information

Time: MWF 9:30-10:20 am.

Location: Nord Hall 356

Instructor: Peter Thomas, Yost 212, tel. 368-3623 (Dept. office 368-2880), case ID = pjt9 (`email`: caseID/at/cwru.edu)

Texts: The principal text for the course will be *Differential Dynamical Systems* by J.D. Meiss (2007, published by SIAM Press[1]). An excellent (and inexpensive) supplement is *A Second Course in Elementary Differential Equations* by P. Waltman; published by Dover press. This book is also required. Although numerical solution of ODEs is not the focus of the course (*cf.* MATH 432), some simulations will help with intuition. Students may use any software package for solving differential equations (matlab, mathematica, xpp). However, the following text is recommended: *Simulating, analyzing, and animating dynamical systems: a guide to XPPAUT for researchers and students*, by G.B. Ermentrout; published by SIAM Press. Some excerpts will be provided from other sources as well.

Prerequisites: A first course in elementary differential equations (e.g., MATH 224 or 228) is required. A previous course in linear algebra (MATH 201, or 307) is strongly recommended, as is a first course in analysis (MATH 321).

Description: Differential equations provide the fundamental mathematical language for quantitatively describing dynamical systems in a vast range of natural sciences, including both the physical sciences (physics, chemistry, geology), the biological sciences (from neuroscience and cell biology to ecology and evolution), and recently even the social sciences (particularly economics). MATH 435 is a second course in *ordinary* differential equations (ODEs), *i.e.* equations of the form

$$dx/dt = f(x, t)$$

where x is a vector in $\mathbb{R}^n$ and f is a function giving the rate of change of x in time.

The course covers topics such as: existence, uniqueness, and continuation of solutions of ODE; linear systems, fundamental matrix, qualitative methods (phase plane); dependence on initial data and parameters (Gronwall's inequality, nonlinear variation of parameters); stability for linear and nonlinear equations, linearization, Poincare-Bendixson theory. Additional topics may include regular and singular perturbation methods, autonomous oscillations, entrainment of forced oscillators, and bifurcations.

Grading: There will be two midterm exams and a final exam. Each midterm will determine 15% of the course grade, and the final will determine 25%. The remainder will be based on homework assignments (30%) and a course project (15%).

Office Hours: MWF 8:30-9:20 am, Yost 212.

Homework: As noted in the section on grading, your homework will determine a significant portion of your grade in this course. There will be an assignment due every two weeks, more or less. Many of the problems will require some thought, so don't wait until the last minute to start working on an assignment. *Late work will be penalized.*[2] You are permitted to discuss the problems with one another, but any work you turn in must be your own. If you find yourself having difficulty with the problems, please meet with the instructor.

Workload: The workload for this course should average roughly 10-15 hours per week. If you find the assignments taking considerably more time (or less) please inform the instructor.

[1] SIAM (The Society for Industrial and Applied Mathematics) offers a 30% discount to members. Students can join for free! See `http://www.siam.org/membership/individual/free.php`

[2] Homework turned in t days late which would have earned a score of s_0 if turned in on time will instead be given a score of $s(t)$, where $s(t)$ is the solution of the linear equation $ds/dt = -s/\tau, s(0) = s_0, \tau = 1$ week.

Additional References:

V.I. Arnold, *Ordinary Differential Equations*
G. Birkhoff, G-C. Rota, *Ordinary Differential Equations*
J. Hale, H. Koçak, *Dynamics and Bifurcations*
M. Hirsch, S. Smale and R. Devaney, *Differential Equations, Dynamical Systems & an Introduction to Chaos* (Second Edition).
D.W. Jordan and P. Smith, *Nonlinear Ordinary Differential Equations* (Fourth Edition).

More advanced than the above:

J. Guckenheimer, P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*
J. Hale, *Ordinary Differential Equations*
P. Hartman, *Ordinary Differential Equations*
Y. A. Kuznetsov, *Elements of Applied Bifurcation Theory*
J. Murray, *Asymptotic Analysis* (includes material on singular perturbation theory for ODEs)

List of Topics (Approximate)

1. Overview (Meiss Ch.1 / supplement from Birkhoff & Rota, *Ordinary Differential Equations*, §1)
2. Linear ODEs, including the matrix exponential (Meiss §2 / Waltman §1)
3. Existence and uniqueness of solutions of ODEs, including the contraction mapping theorem (Meiss §3 / Waltman §3)
4. Qualitative analysis of nonlinear ODEs, including stability, topological conjugacy, and the Hartman-Grobman theorem (Meiss §4 / Waltman §2 & supplements)
5. Invariant manifolds, including center manifold analysis (Meiss §5)
6. Singular perturbation methods (supplement from Jordan & Smith, Nonlinear Ordinary Differential Equations, §6)
7. Phase plane analysis (Meiss §6, selected)
8. Bifurcation theory (Meiss §8, selected – time permitting)

Note: Prof. Thomas will be traveling until September 1. The first course meeting will be Friday, September 2. Makeup classes will be scheduled in consultation with the class.

Detailed List of Topics and Approximate Schedule

Friday 2016-09-01 .

Topic Solution of linear and nonlinear equations: selected special cases.

Reading Birkhoff & Rota, Chapter 1 (Bb link). Birkhoff and Rota give an overview of "elementary" ODEs in their first chapter. Ignore the comments about numerical solution (*e.g.* on page 22, where they write "Many computers have only 7-digit accuracy". The first edition appeared in 1959 and the fourth edition in 1989; the basic theorems haven't changed, though.) The rest of the chapter is a gold mine of ideas.

Exercises

1. B&R Exercises 1-D-2. Express in closed form all solutions of the following DEs:
   (a) $y' = (x^2 - y^2)/(x^2 + y^2)$
   (b) $y' = \sin(y/x)$.

Solution of linear equations. Meiss §1.

Wednesday 2016-09-06 .

Topic Solution of linear equations.

Reading Meiss §1.

Exercises

1.

Date Date .

Topic (topic)

Reading (reading)

Exercises

1.