Quasicycles in the stochastic hybrid Morris-Lecar neural model

Heather A Brooks, University of Utah, heather@math.utah.edu
Paul C Bressloff, University of Utah, bressloff@math.utah.edu

Intrinsic noise arising from the stochastic opening and closing of voltage-gated ion channels has been shown experimentally and mathematically to have important effects on a neuron's function. Study of classical neuron models with stochastic ion channels is becoming increasingly important, especially in understanding a cell's ability to produce subthreshold oscillations and a response to weak periodic stimuli. While it is known that stochastic models can produce oscillations (quasicycles) in parameter regimes where the corresponding deterministic model has only a stable fixed point, little work has been done to explore these connections to channel noise. Using a stochastic hybrid Morris-Lecar model [1, 2, 3, 4], we combine a system size expansion in K^+ and a quasi-steady state (QSS) diffusion approximation in persistent Na^+ in order to derive an effective SDE. (The QSS analysis exploits the fact that persistent Na^+ channels are fast but the number of such channels is small). By determining the corresponding power spectrum, we establish that noise significantly extends the parameter regime in which subthreshold oscillations occur. Moreover, we find that under physiological conditions, the major contributor to the existence of quasicycles is persistent sodium channel noise.

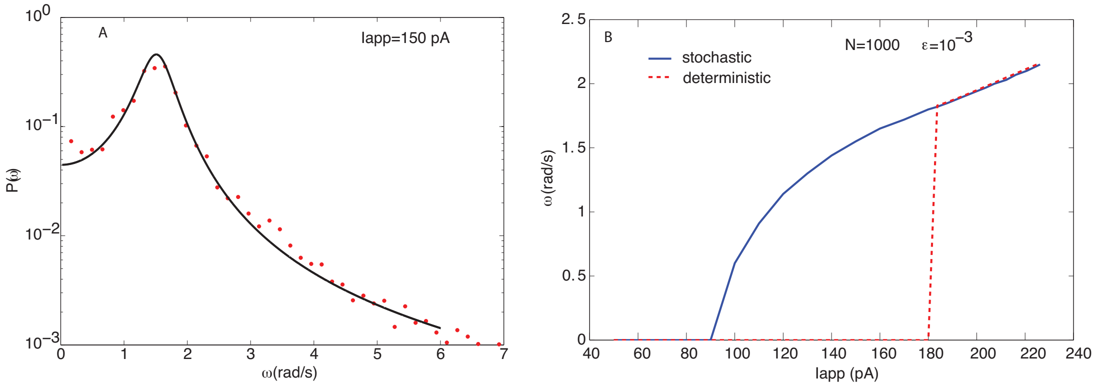

Figure 1: (A) The power spectrum of the voltage in the stochastic hybrid Morris-Lecar model below supercritical Hopf bifurcation point I^*_{app}. The spectrum has a well-defined peak around the Hopf frequency, $\omega_c = 1.51$ rad/s, indicating the presence of oscillations (quasicycles). Red asterisks from numerical simulations via the Gillespie algorithm, whereas the solid (black) line is the analytical prediction. (B) Channel noise increases the range of applied current values for which subthreshold oscillations exist. This also increases the range of frequencies which the model may produce.

References

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