Renewal approach to the stability analysis of noisy spiking recurrent networks

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The classical approach to the analysis of the stability of integrate-and-fire network of neurons is to apply Fokker-Planck equation for the voltage distribution with boundary conditions. This approach is highly successful for network consists of one state variable neurons with static couplings. Nonetheless, the Fokker-Planck stability analysis is considerably difficult to be extended for the network of neurons with more than one state variable (e.g. networks with the dynamic synapse or correlated input), due to highly non-trivial boundary conditions. To overcome these problems, we present an alternative approach: instead of studying the evolution of the voltage distribution, we analyze the system by studying how its self-consistent inter-spike interval density evolves. This determines evolution of the firing rate, which determines the recurrent input to close the system. This approach also allows to determine the eigenvalue-equation of the perturbed system. Here, we recover the same eigenvalue equation as the classical Fokker-Planck approach for stability condition of a network with static synapses [2]. Furthermore, we show the analysis can easily be extended to networks with more state variables. As a computationally interesting example, we illustrate phase transition of a recurrent network of integrate-and-fire neurons with the Short-Term-Depression (STD). Using this formalism, we demonstrate that the stability condition of such a network can be effortlessly analyzed. Here, much insight is gained from our method that activity dependent recurrent couplings contribute to the phase transition of the system from the asynchronous state to collective dynamics of population spikes (i.e. synchronization) in the presence of external iid noise. Our results here provide a spiking based theory for the previously known phenomena of populations spike [3] achieved in a rate model. Additionally, this analysis revels the bifurcation is indeed of Hopf type and provides a numerical evidence on the super-critical nature of this phase transition. The method also allows for simulation of the network with STD in its thermodynamical limit. The numerical simulations demonstrate a full agreement with our linear stability analysis.

The complete account of the method and its detailed analysis is available at [1].

References

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