STOCHASTIC MODELING OF NEURONAL FIRING ACTIVITY BY GENERALIZED ORNSTEIN-UHLENBECK PROCESSES

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The classical Leaky Integrate-and-Fire (LIF) neuronal model ([1] and references therein) is widely used to model the voltage of the neuronal membrane. In the last two decades many adaptations and specializations (see, for instance, [2],[3]) are applied to the original stochastic differential equation on which it is based, sometimes also criticisms have been highlighted ([4]). We stress the robustness of this model and its adaptability to the phenomenological evidences when the involved parameters can be assumed time-dependent. In particular, we consider the following LIF-type stochastic differential equation including time-dependent functions, for $t \geq t_0$,

$$dV = \{-\alpha \left[V - \rho(t)\right] + I(t)\varphi(t)\}\, dt + \sigma(t)dW, \quad V(t_0) = v_0. \tag{1}$$

Under hypotheses of regularity on the coefficient functions of (1), the diffusion process $V(t)$ with infinitesimal moments

$$A_1(v,t) = -\alpha \left[v - \rho(t)\right] + I(t)\varphi(t), \qquad A_2(t) = \sigma^2(t) \tag{2}$$

is solution of (1) and is also Gaussian. Recalling that the Ornstein-Uhlenbeck (OU) process solves the standard LIF equation, and due to the presence of time-dependent functions in the infinitesimal moments (2), we refer to the process $V(t)$ as generalized OU process ([5]-[7]). In (1) the parameter α is related to the characteristic time of the membrane potential, v_0 is the initial (reset) value, $\sigma(t)$ represents a varying intensity of noise and W is the standard Brownian motion. Furthermore, the function $\rho(t)$ represents the time-varying equilibrium potential, $I(t)$ can play the role of a superimposed input current or a synaptic current originated from surrounding neuronal activity, $\varphi(t)$ is a particular detector function suitably specified to describe different aspects of the neuronal activity.

The first passage time (FPT) through specified thresholds of the above processes models the spike time of the neuron. Some of ours theoretical, numerical and asymptotic results about such FPTs are of particular interest in this context. Several phenomena, such as interactions between neurons, adaptation of the firing activity, effects of input currents, occurrence of spike trains can be modeled for suitable choices of the involved time-dependent functions.

References

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