A Leaky Integrate-and-Fire neuronal model with Gamma distributed interspike intervals.

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One of the most popular neuronal models to describe the generation of the Interspikes Intervals (ISIs) distribution is the Leaky Integrate-and-Fire (LIF) concept. In one of its variants and in absence of external inputs, the membrane potential evolution is described by an Ornstein-Uhlenbeck (OU) stochastic process $X = \{X_t, t \geq 0\}$, i.e., a stochastic diffusion process which is a solution of the stochastic differential equation

$$dX_t = \left(-\frac{X_t}{\theta} + \mu\right) dt + \sigma dW_t, \tag{1}$$

where $\theta > 0$ accounts for the spontaneous decay of the membrane potential, μ and $\sigma > 0$ characterize the neuronal input and its variability and W_t indicates the standard Brownian motion. The distribution of the first passage time (FPT) T of this process through a constant boundary S is commonly used to model the ISI distribution. After each spike the membrane potential is reset to zero making the ISIs identically distributed and independent random variables.

Experimentally recorded ISIs often fit well the Gamma distribution and this distribution is also often used for modelling purposes ([2]). This fact suggests to ask if and under which conditions the diffusion LIF model (1) can generate Gamma distributed ISIs. It is hardly possible to expect that the input μ is time-dependent but identical for each ISI, thus we search the answer by investigating either random reset after spike generation or time-dependent firing threshold $S(t)$.

The technique developed to solve the inverse FPT problem is applied. In the FPT problem the unknown quantity is the FPT distribution while the data are the diffusion process with its parameters, the initial value and the boundary value (eventually time depending). In the typical inverse FPT problem the unknown is the boundary shape while the diffusion process, its parameters and the FPT distribution are assigned. Algorithms to solve the inverse FPT problem are known ([1],[3]). An alternative to searching for the threshold in the inverse FPT problem is looking for the distribution of the initial value of the process. In this second case the process, the boundary and the FPT distribution are assigned.

In this talk we propose a method to solve this new inverse FPT problem. Then we apply both inverse FPT problems to LIF model (1). A set of examples are discussed in correspondence to different choices of the parameters of the Gamma distribution.

References

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