Cosmological Constraints on Anisotropic Thurston Geometries

Ananda Smith, Craig Copi, Glenn Starkman

Department of Physics/CERCA/ISO, Case Western Reserve University, Cleveland, Ohio, 44106

Introduction to Anisotropic Curvature

- Much of modern cosmology relies on the Cosmological Principle, which assumes that the Universe looks the same at every position (**homogeneity**) and in every direction (**isotropy**).
- To test the Cosmological Principle, we examine models whose spatial geometries are homogeneous but **anisotropic** -- i.e., have direction-dependent spatial curvature.
- There are five homogeneous and anisotropic 3D geometries: $S^2 \times \mathbf{R}$, $\mathbf{H^2} \times \mathbf{R}$, the universal cover of $U(\mathbf{H^2})$, Nil, and Solv. They are known as the **anisotropic Thurston geometries**.

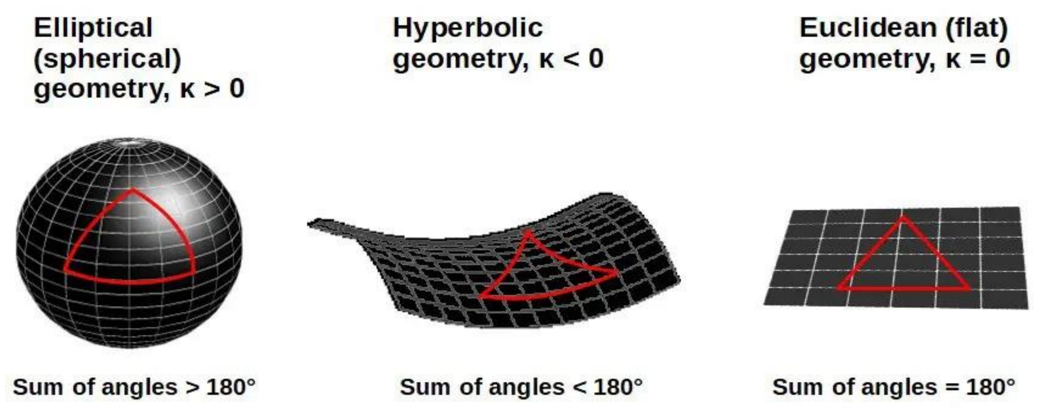


Figure 1 from [1]. The sphere, hyperboloid, and plane have positive, negative, and zero curvature, respectively. They are 2D analogs of what the Cosmological Principle assumes our Universe is like–homogeneous and isotropic. The five anisotropic Thurston geometries have direction-dependent curvature. For instance, in $S^2 \times \mathbf{R}$, two directions have positive curvature, while the remaining direction is flat.

The Cosmic Microwave Background

- The cosmic microwave background (CMB) is the oldest light in the Universe. It is nearly a blackbody and has a **very uniform temperature** with tiny fluctuations commonly attributed to density perturbations present when the CMB formed.

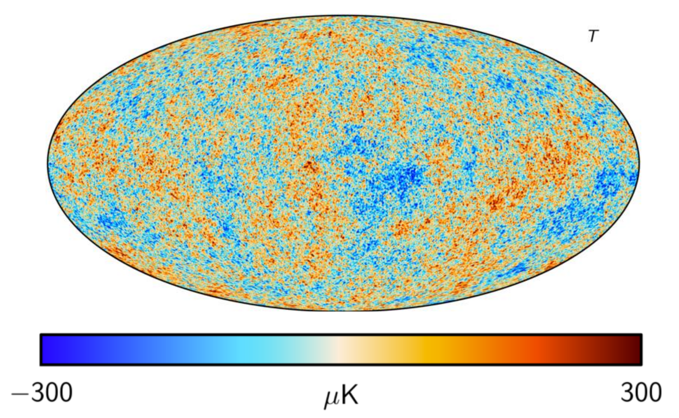


Figure 2: Temperature fluctuations in the CMB. Image courtesy of *Planck* collaboration [2].

- As light travels through a Universe with anisotropic curvature, it is stretched differently in different directions. This **induces fluctuations in the observed CMB temperature**, independent of density perturbations.
- **We compute the induced CMB temperature fluctuations in Universes with anisotropic Thurston geometries**. We place constraints on these spaces by imposing that the induced fluctuations cannot be more powerful than those that are observed in our CMB.

Step 1: Solve Einstein's Field Equations

- To compute the induced CMB temperature fluctuations, we must first determine how Universes with anisotropic curvature evolve in time by solving Einstein's field equations (EFEs):

$$G^{\mu}{}_{\nu} = 8\pi G T^{\mu}{}_{\nu}$$

Spacetime curvature — Mass-energy

- We solve for the **scale factors**, which describe how space expands over time.
- The solutions to the EFEs depend on the mass-energy in our Universe. We choose a mass-energy assumed in the standard model of cosmology: **perfect fluid matter and cosmological constant** ("dark energy").
- From the EFEs, we define a dimensionless curvature parameter Ω_K. **Each geometry can be made arbitrarily close to flat by taking its Ω_K to closer zero**.
- We find that the EFEs demand there be two independent scale factors in every anisotropic Thurston geometry, meaning **different directions expand at different rates**. Consequently, photons undergo direction-dependent **redshifts.**

Step 2: Compute CMB Temperature

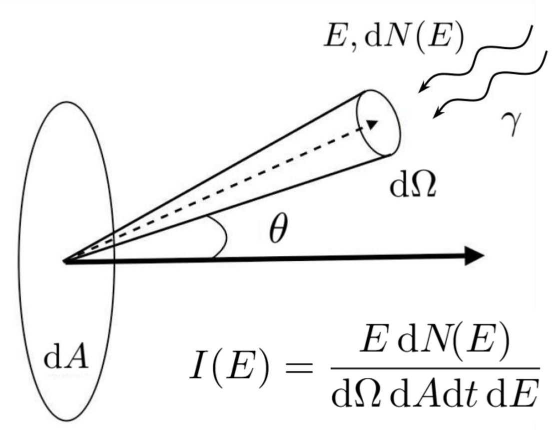


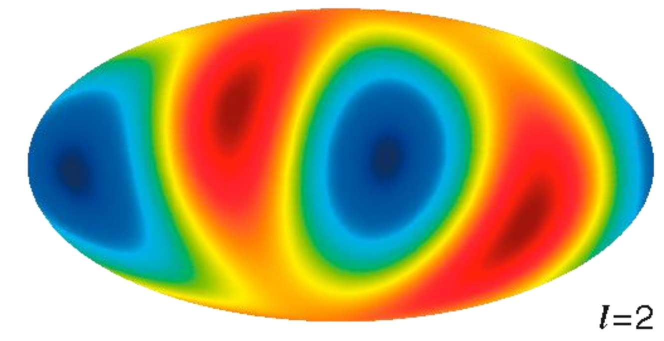
Figure 3: Adapted from Figure 1 of [3]. A source subtending a solid angle dΩ emits $dN(E)$ photons in the energy band $E + dE$. These photons transfer energy to a detector surface with area dA over a time dt. The specific intensity I(E) characterizes this radiative transfer.

- The CMB is a blackbody, so we determine its temperature from measurements of its **specific intensity**, which characterizes the transfer of energy from photons as they hit a detector surface.
- We assume the CMB's specific intensity when it forms is a blackbody spectrum with a uniform temperature.
- As a consequence of Liouville's theorem, the specific intensity we measure today is simply a **blackbody with a direction-dependent temperature.**
- The induced temperature fluctuation ΔT is a pure Y_{20} spherical harmonic (a **quadrupole**) in all five geometries.

$$\Delta T \equiv T - T_0 \approx -\frac{1}{2}\Omega_K K^{(3)}(3\cos^2\theta - 1)T_0 F(\eta_r) = -2\sqrt{\frac{\pi}{5}}K^{(3)}F(\eta_r)T_0\Omega_K Y_{20}(\theta, \varphi).$$

Step 3: Constraints on Curvature

Figure 4 courtesy of [5]. In Universe with anisotropic curvature, a quadrupolar fluctuation (like the one pictured right) is induced in the observed CMB temperature. Since this induced quadrupole must not be any more powerful than the observed CMB quadrupole, Ω_K is strongly constrained in the five anisotropic Thurston geometries.



Induced quadrupole power:

$$D_2 \approx \frac{12}{25}(K^{(3)})^2 F(\eta_r)^2 T_0^2 \Omega_K^2$$

≤

Observed quadrupole power [4]:

$$D_2^{\text{obs}} = 225.9\,\mu\text{K}^2$$

$$|\Omega_K| \lesssim \frac{\left(\frac{25}{12}D_2^{\text{obs}}\right)^{1/2}}{|K^{(3)}F(\eta_r)|T_0} \lesssim 10^{-5}$$

Summary

- We examine Universes with spacetimes that are homogeneous but anisotropic. This amounts to considering the five anisotropic Thurston geometries.
- With perfect fluid stress energy, these Universes must expand differently in different directions to satisfy EFEs.
- A quadrupolar fluctuation in the observed CMB temperature is induced in all five of these spaces.
- For the power in the induced quadrupole to be no greater than the power in the observed quadrupole, $|\Omega_K| \leq 10^{-5}$ in each geometry. **This stringently limits the cosmological viability of models with anisotropic curvature.**

References

[1] C. Bunney and V. Hirovonen. "The Friedmann Equations Explained: A Complete Guide."

[2] Planck Collaboration et. al. (2020). "Planck 2018 results. IV. Diffuse component separation." *Astronomy and Astrophysics*, vol. 641, 1-74.

[3] D. Forgan. (2009). "An Introduction to Monte Carlo Radiative Transfer."

[4] Planck Collaboration et. al. (2020) "Planck 2018 results. VI. Cosmological parameters." *Astronomy and Astrophysics*, vol. 641, A6, 1-67.

[5] C. Bennet et. al. (2011). "Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Are There Cosmic Microwave Background Anomalies?" *The Astrophysical Journal Supplement Series*, vol. 192, 1-19.

Check out our paper on the arXiv here:

This work was supported by NASA ATP Grant RES240737.